Bayesian Data Analytics for Reliability Modeling Improvement

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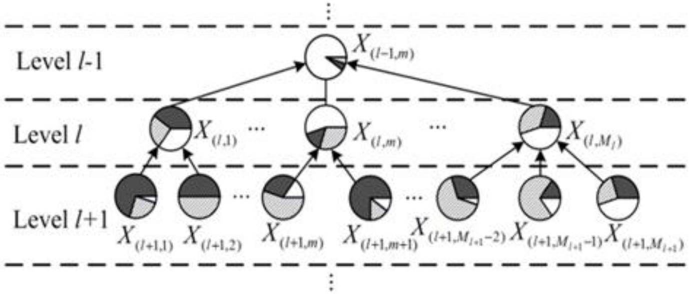
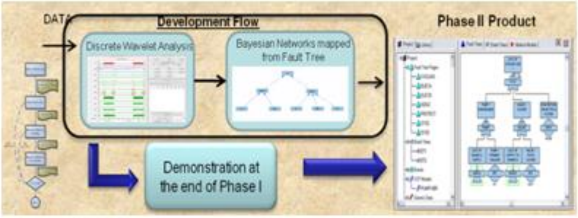
Some of Current and Former Team Members in DSSI Lab at Dept. of IMSE:

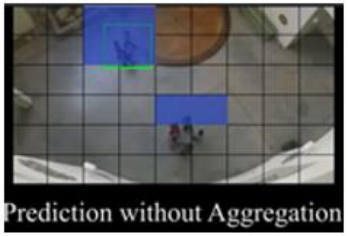
Research Expertise and Highlights:

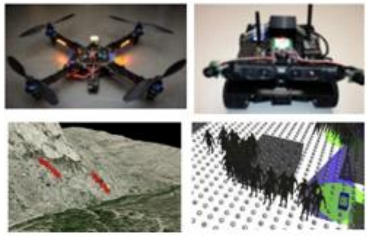
- Multi-level Data Integration and Analytics for Mission Critical System Reliability Assessment, Testing, Diagnosis, Prognosis and Real-Time Health Management.





- Multi-fidelity Data Integration and Analytics for Crowd Surveillance Improvement.





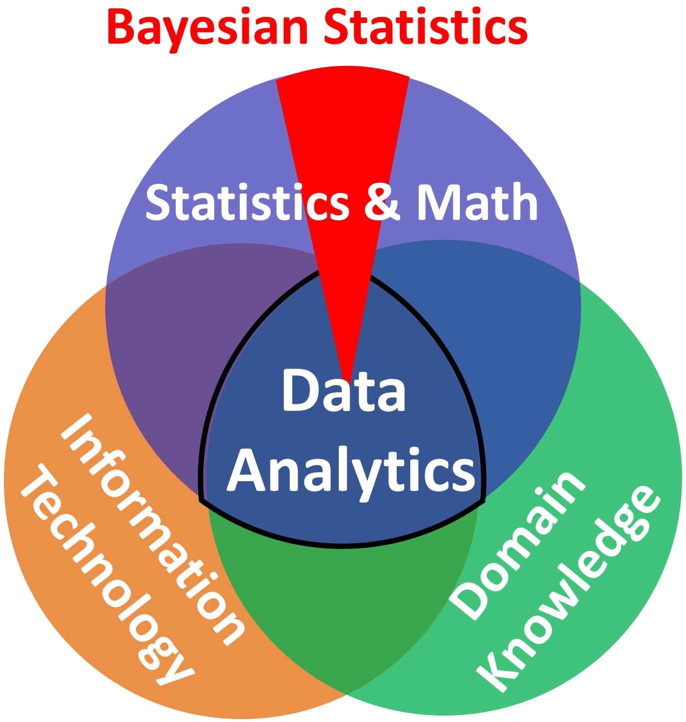
- Various Data Science and System Informatics Methods Development and Diverse Applications.

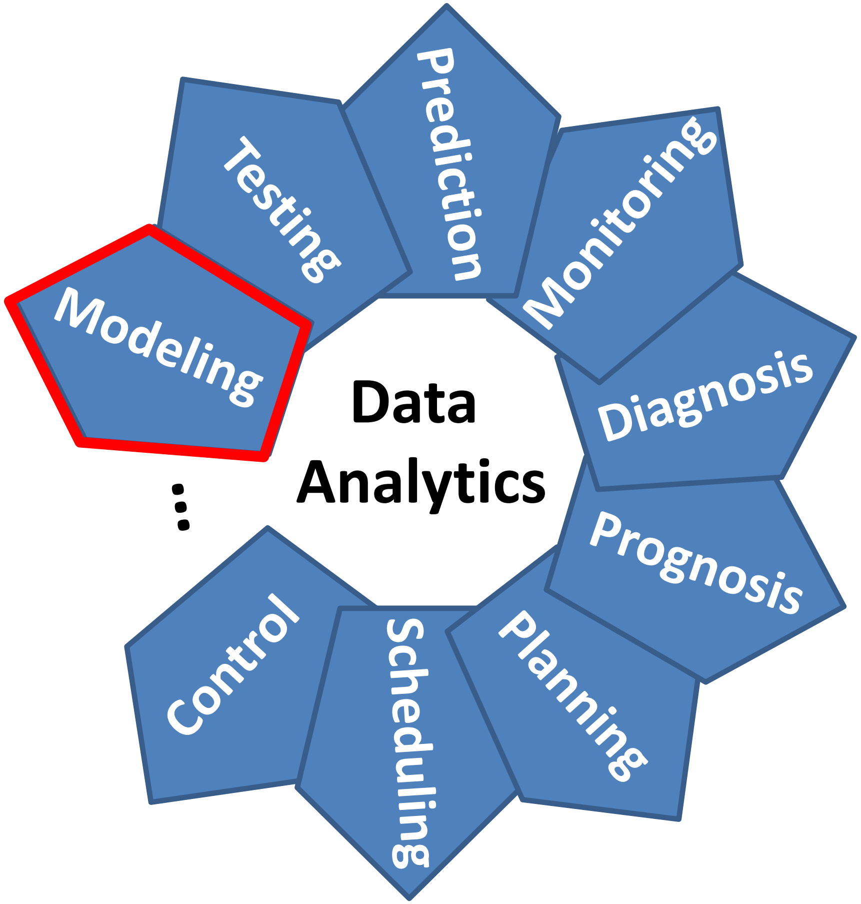
Quality & Reliability · Healthcare · HVAC · Wind energy · Water …

Outline

- **Background**
- **Part I - Multi-level Data Fusion**
- **Part II - Heterogeneous Data Quantification**
- **Summary**

Data Analytics





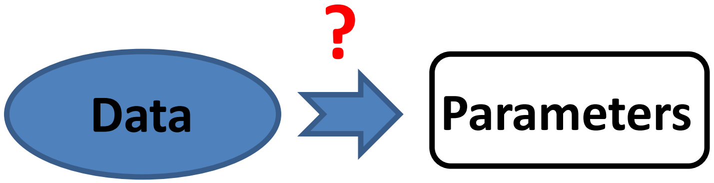
- **Focus:**

Bayesian Data Analytics for Reliability Modeling Improvement

Key Word: Bayesian

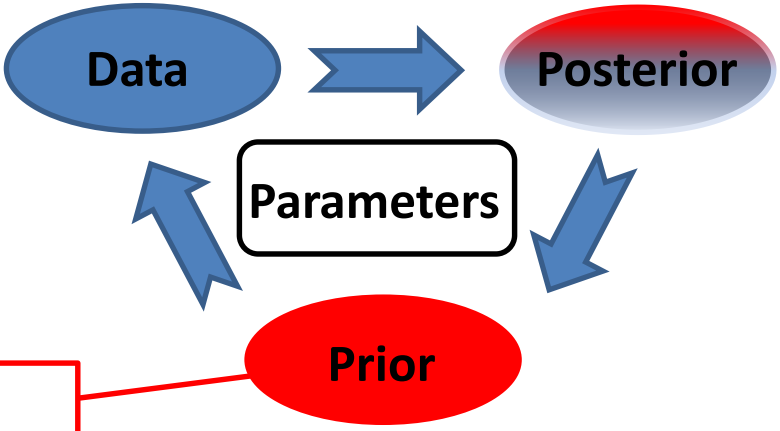
- **Parameter Learning**

Classic Statistics



Limited Data or No Data

Bayesian Statistics



- External data sources
- Domain knowledge
- Non-informative prior

……

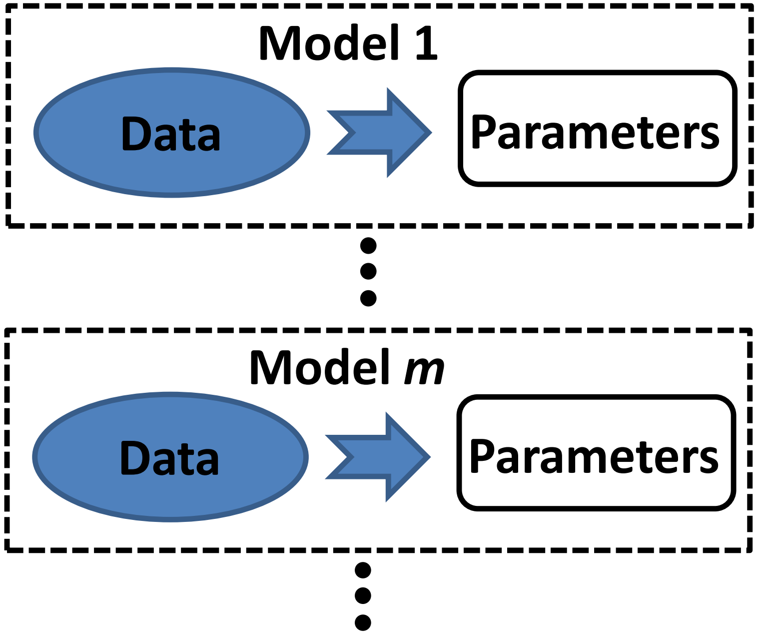
Flexible & Coherent

Methodology I:
Multi-level Data Fusion

Key Word: Bayesian (Cont'd)

- **Model Learning**

Classic Statistics



Model 1

Data → **Parameters**

⋮

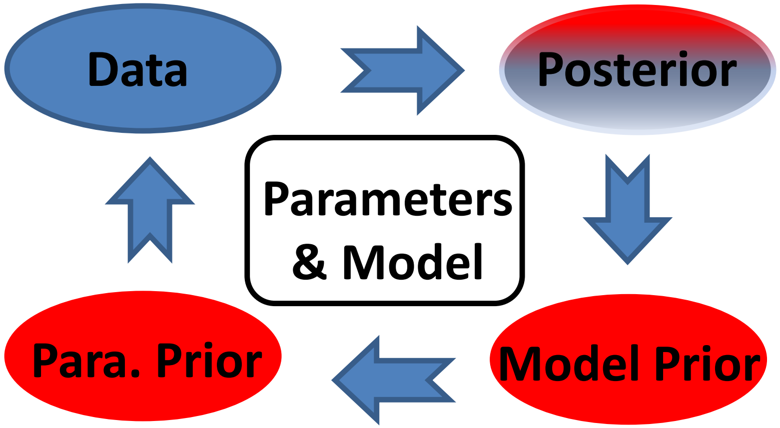
Model *m*

Data → **Parameters**

⋮

- **Underfitting/Overfitting**
- **Inefficient**

Bayesian Statistics



Data → **Posterior** → **Model Prior** → **Para. Prior** → **Data**

Parameters & Model

Efficient & Effective

Methodology II: Heterogeneity Quantification

Key Word: Reliability Modeling

- Reliability: product quality over time[1]

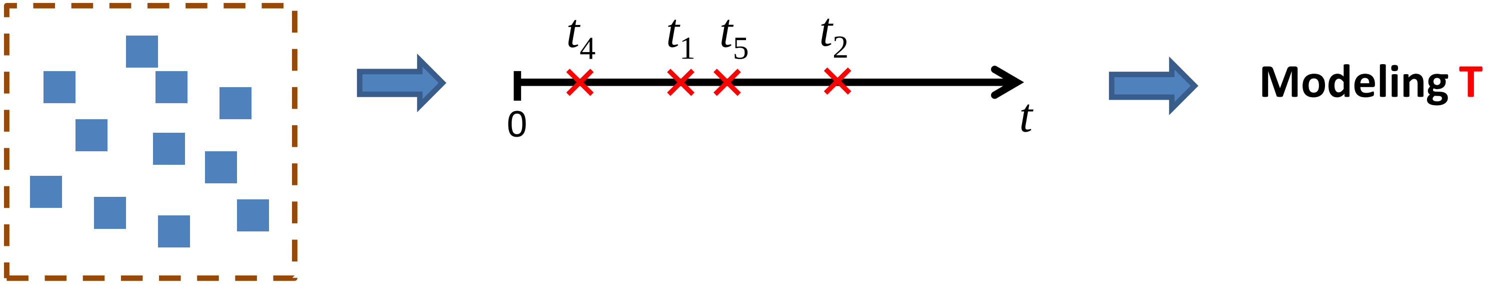
Pr(T>t)

Time-to-failure

- Reliability modeling

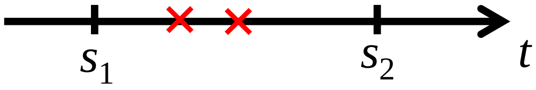
Product Sample:

Reliability Data:



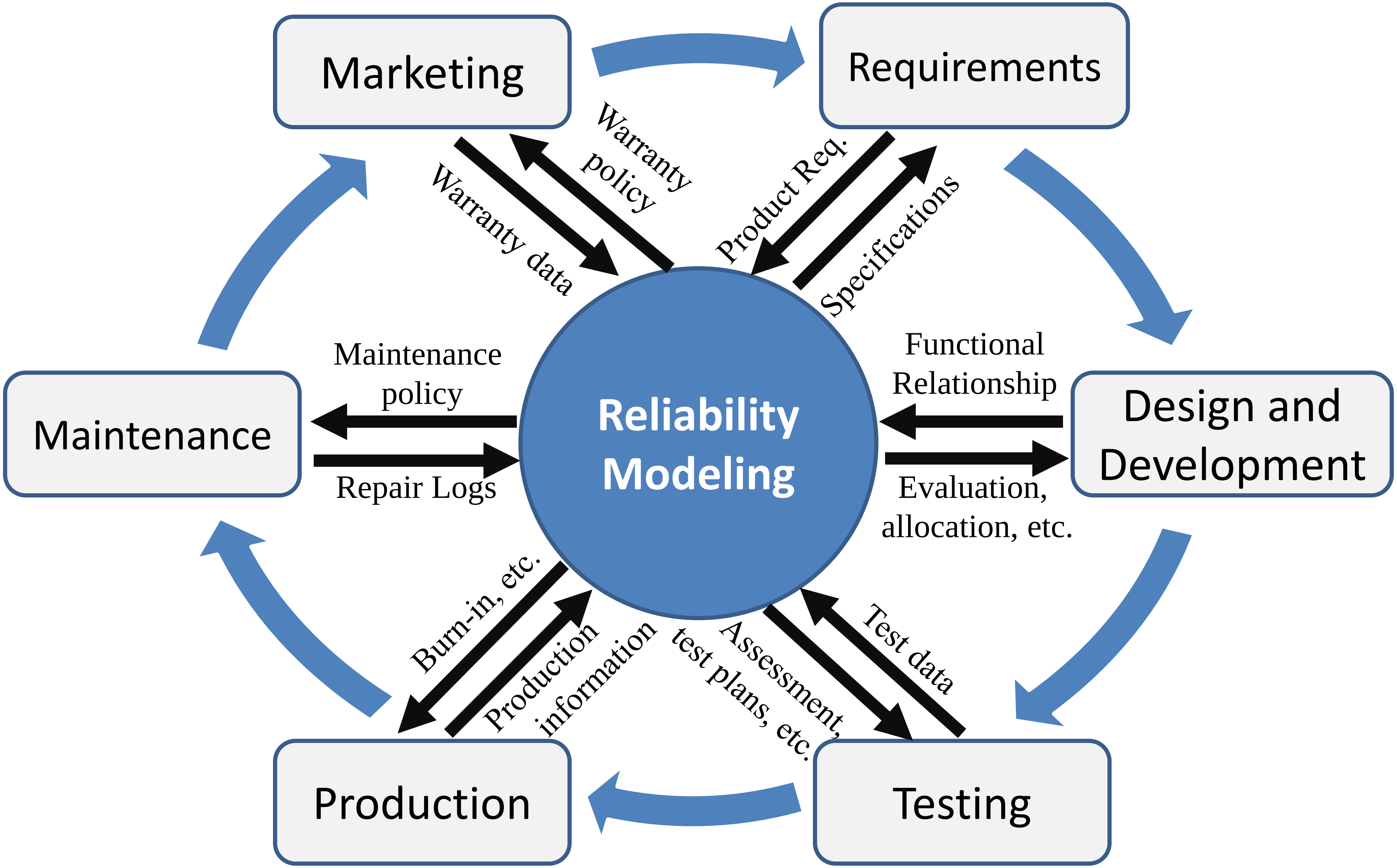
Modeling T

- Data Feature
  - **Non-negative and asymmetric**
  - **Censoring**



  - **Covariates**
  - **Others: availability, heterogeneity**, etc.

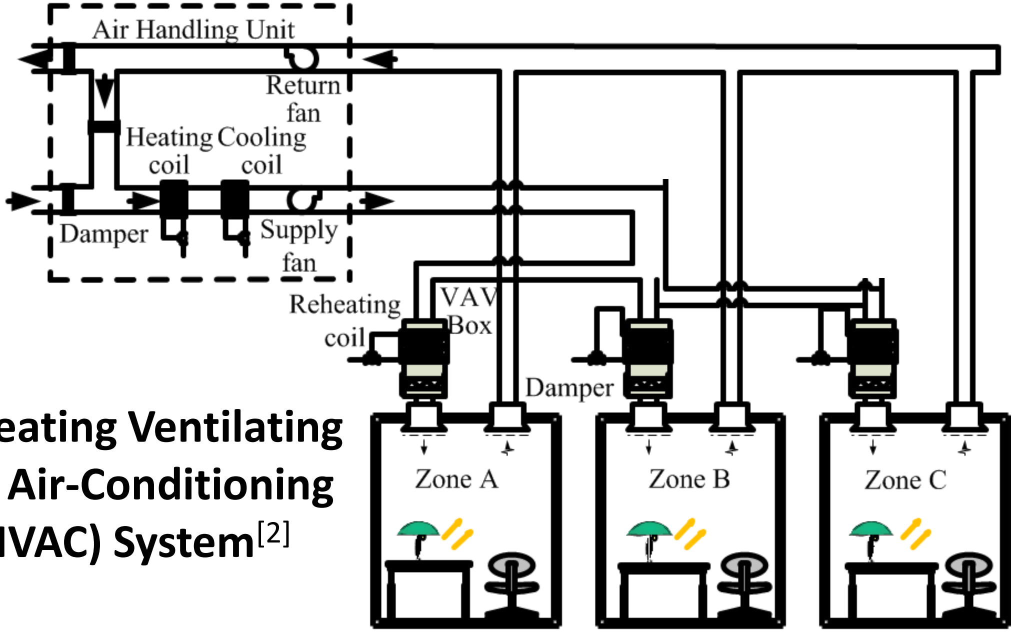
Lifecycle View of Reliability Modeling



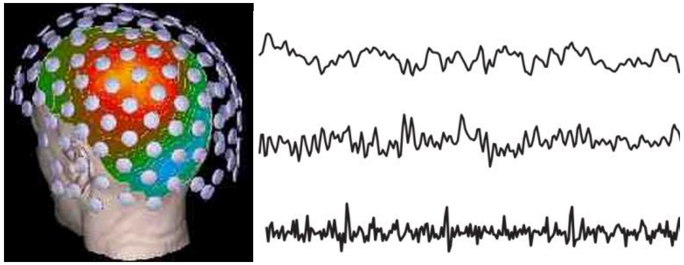
Part I - Multi-level Data Fusion:

Bayesian Multi-level Information Aggregation for Hierarchical Systems Reliability Modeling Improvement

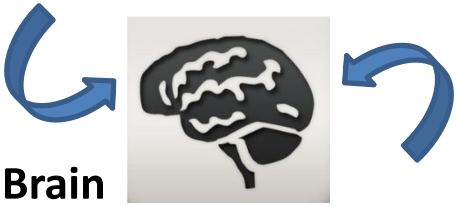
Vision



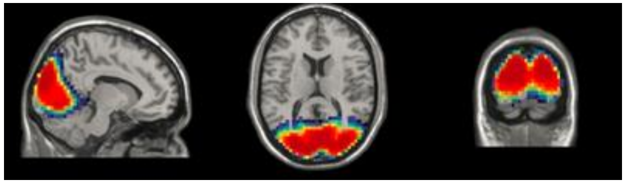
Heating Ventilating & Air-Conditioning (HVAC) System[2]



EEG/MEG (high-temporal-resolution)[3]



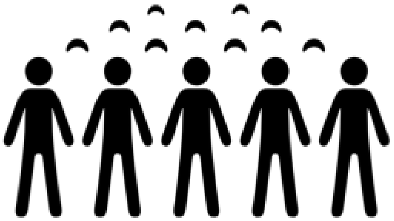
Brain



fMRI (high-spatial-resolution)[3]

Crowd Surveillance System[4]

Unmanned Ground Vehicle (UGV)



Crowd

Unmanned Aerial Vehicle(UAV)

Data-rich Environment:

Data Fusion

Focus: System Reliability

- **Performance index**: system reliability
- **Modeling Challenges:**
  - Expensive system-level tests
  - Scarce/absent engineering knowledge
  - Complex failure relationship
  - High requirement on reliability assessment

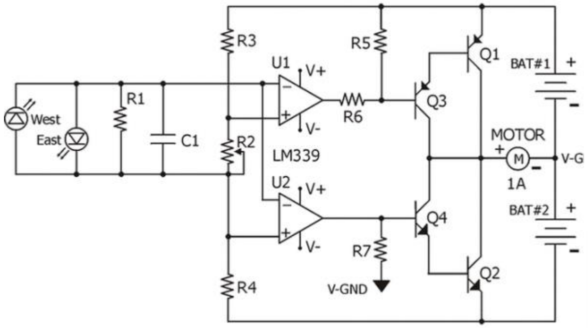
Missile
($\$10^3$k - $10m)

- **Research Goal:**

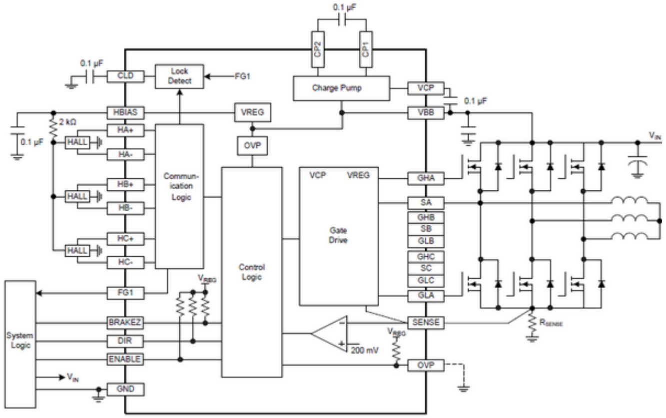
Improve system-level reliability modeling by utilizing all reliability information throughout the system in a **systematic** and **coherent** manner.

Opportunity I: Hierarchical System Structure

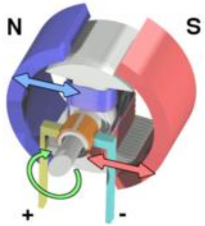
Electro-Mechanical-Actuator (EMA) System



Power Supply (PS)



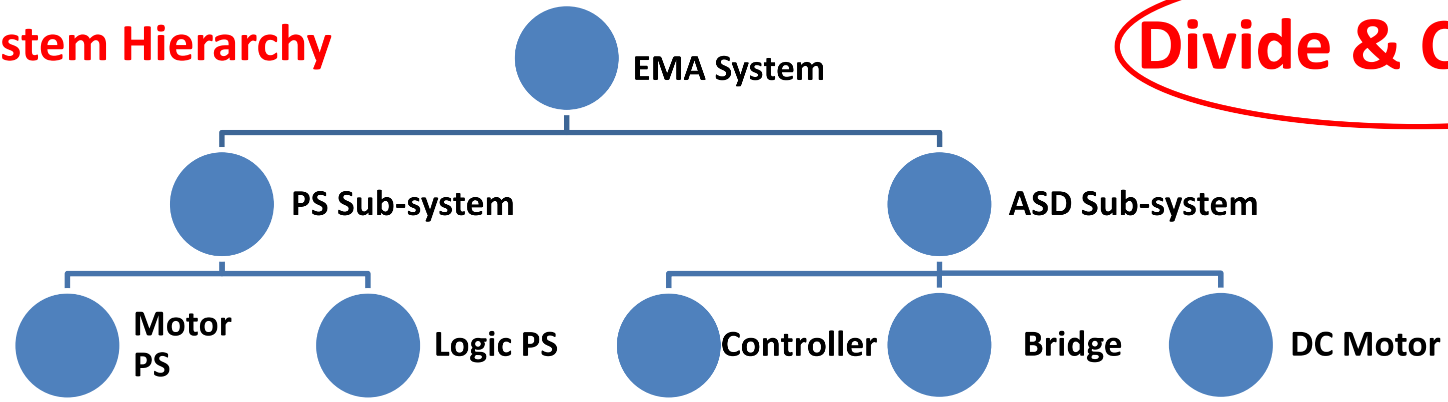
Actuator Servo Drive (ASD)



DC Motor

Elements in System Hierarchy

Divide & Conquer



EMA System

PS Sub-system

ASD Sub-system

Motor PS

Logic PS

Controller

Bridge

DC Motor

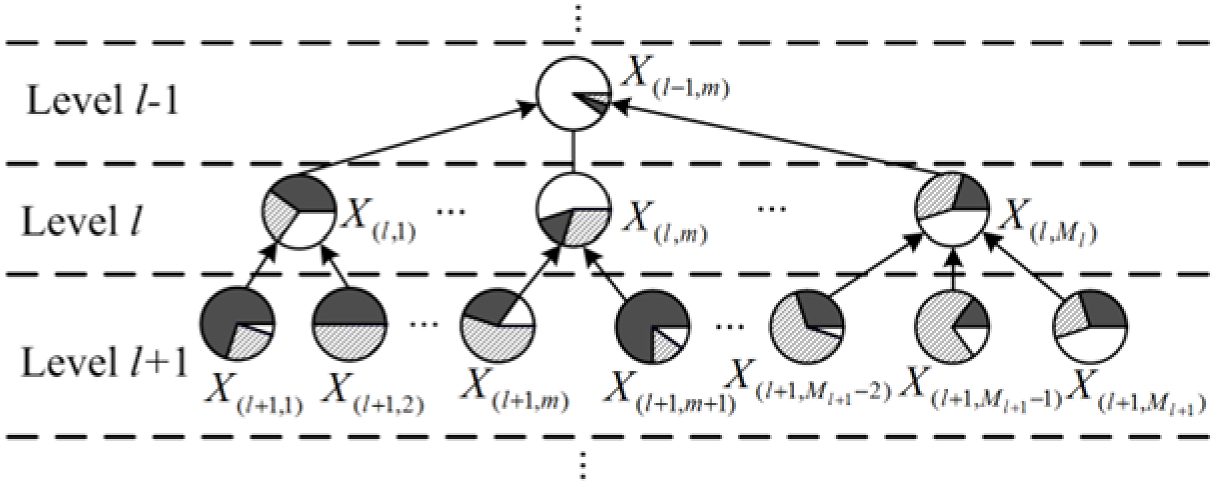
Opportunity II: Multi-source Multi-level Data

- **Multi-source reliability information**: prior knowledge (e.g., domain knowledge, historical studies, etc.) + ongoing reliability test data.

Elements	Prior knowledge	Reliability Test Data
Lower-level	Familiar	(1) Abundant (2) Limited but easy to collect
Upper-level	Unfamiliar or unknown	(1) Absent (2) Limited and/or expensive/hard to collect

- **Multi-level information imbalance**

Aggregation



Reliability Information:

▨ Prior knowledge

■ Reliability test data

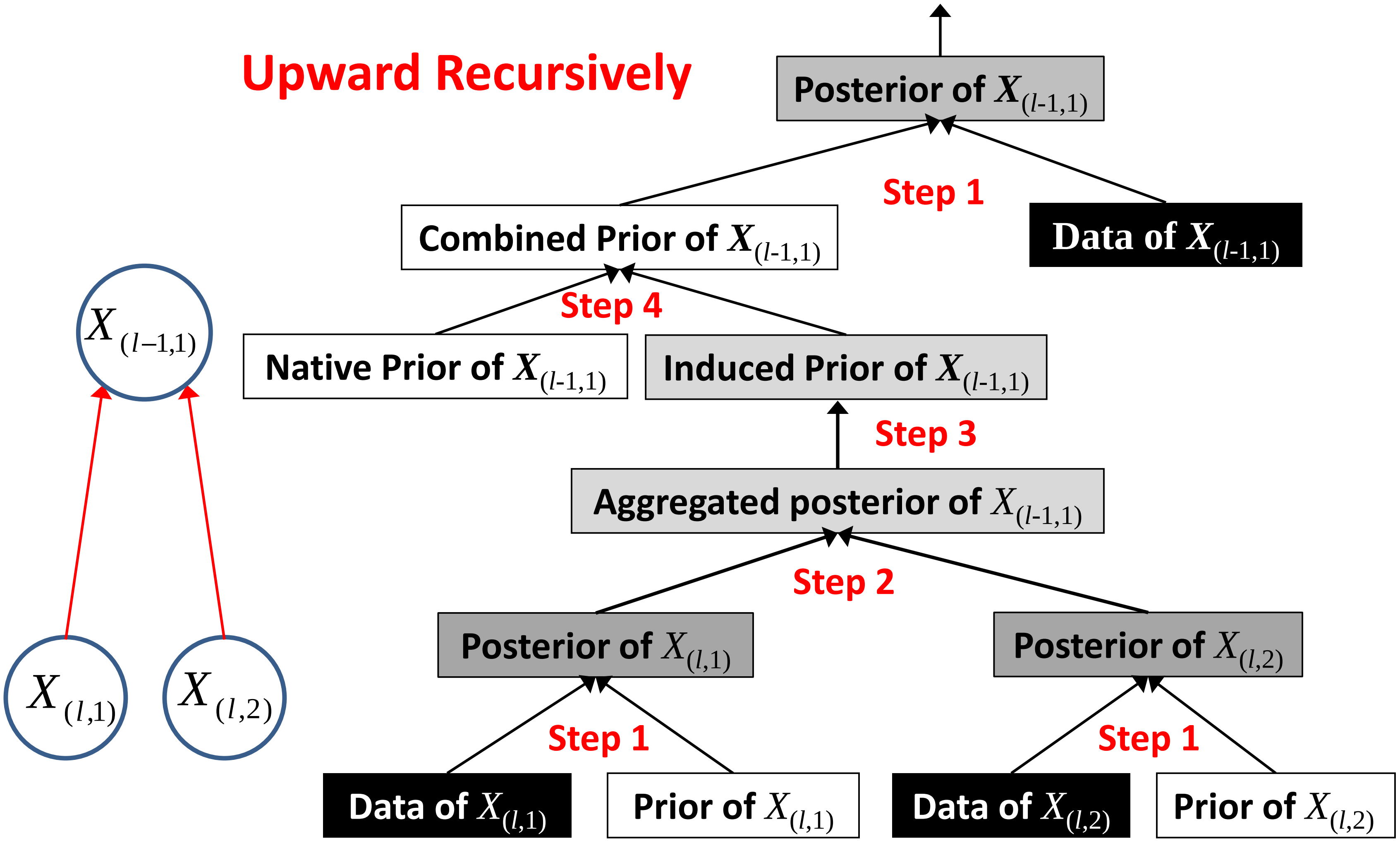
□ Absent information

State of the Art

Methodology Summary		**System Reliability Modeling**	
		Parametric methods	**Semi-parametric/non-parametric methods**
Multi-level information aggregation	**No**	Ramamoorty[5], Camarda *et al.* [6], Cui *et al.* [7], Hoyland and Rausand[8], Coit[9], Jin and Coit[10], Martz and Walker[11], Hamada *et al.* [12], etc.	Klein and Moeschberger[13], Meeker and Escobar (Chapter 3) [14], Ibrahim *et al.* [15]
	Yes	Martz *et al.*[16], Martz and Walker[17], Hulting and Robinson[18]	**To be presented**

- **Features of the proposed model:**
  - Failure-time data with covariates and censoring
  - Semi-parametric modeling
  - Information aggregation from lower levels

Overview of the Proposed Work



Modeling of Individual Element

$X_{(l,k)}$

Proportion Hazard Model (PHM):

Unique

Reliability: $R_{(l,k)}(t) \leftrightarrow H_{(l,k)}(t)$

$$H_{(l,k)}(t) = H^{b}_{(l,k)}(t)\exp\left(\boldsymbol{\beta}^{\mathrm{T}}_{(l,k)}\mathbf{u}_{(l,k)}\right)$$

Covariate

Cumulative hazard function

Baseline cumulative hazard function

Covariate coefficient

Gamma Process Prior:

$$Z(t) \approx G(c_0\alpha(t), c_0)$$

Confidence parameter

Mean function

Multivariate Normal Prior:

$$\pi(\boldsymbol{\beta}_{(l,k_l)}) = N(\boldsymbol{\mu}_{(l,k_l)}, \boldsymbol{\Sigma}_{(l,k_l)})$$

Mean vector

Covariance matrix

Baseline cumulative hazard increments:

$$\Delta H^{b}_{j,(l,k)} \sim Gamma(c_0(\alpha(s_j) - \alpha(s_{j-1})), c_0)$$

Carry information for aggregation

Aggregation Procedure: Step 1

- **Step 1** – Compute the posterior (lower-level element):

Joint Posterior $\propto$ Likelihoods × Joint Priors:

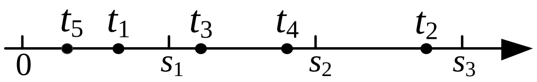
$$p(\boldsymbol{\beta}_{(l,k)}, \Delta H^{b}_{j,(l,k)} \mid \Omega_{(l,k)}) \propto L(\boldsymbol{\beta}_{(l,k)}, \Delta H^{b}_{j(l,k)} \mid \Omega_{(l,k)})\, \pi(\boldsymbol{\beta}_{(l,k)})\, \pi(\Delta H^{b}_{j(l,k)})$$

Integration of data and prior

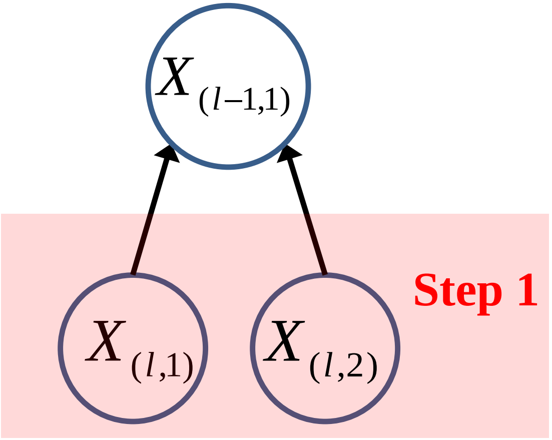
Failure-time data with covariates and censoring

Prior knowledge

Example:



t_n : the actual failure time stamp of test unit n



Bayesian PHM integrates the reliability prior knowledge and failure data

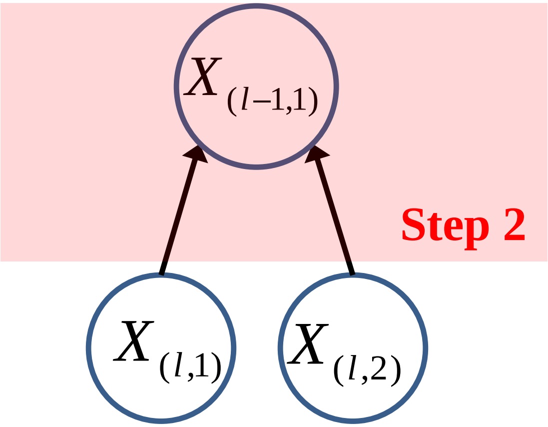
Aggregation Procedure: Failure Relationship

- Failure relationship between two levels:

General Relationships

Reliability functions: $R_{(l-1,k)}(t) = f\left(R_{(l,\kappa)}(t)\right),\ \kappa \in \mathbf{Q}_{(l-1,k)}$

Baseline cumulative hazard increments: ${}^{A}\Delta H^{b}_{j,(l-1,k)} = g(\Delta H^{b}_{j,(l,\kappa)}),\ \kappa \in \mathbf{Q}_{(l-1,k)}$



Example: Series configuration

Aggregated

$${}^{A}R_{(l-1,1)}(t) = R_{(l,1)}(t)R_{(l,2)}(t)$$

$${}^{A}\Delta H^{b}_{j(l-1,1)}(t) = \Delta H^{b}_{j(l,1)}(t) + \Delta H^{b}_{j(l,2)}(t)$$

Information is aggregated through ΔH^{b}_{j} based on failure relationships

Aggregation Procedure: Steps 2-3

- **Step 2** - Aggregate the posterior:

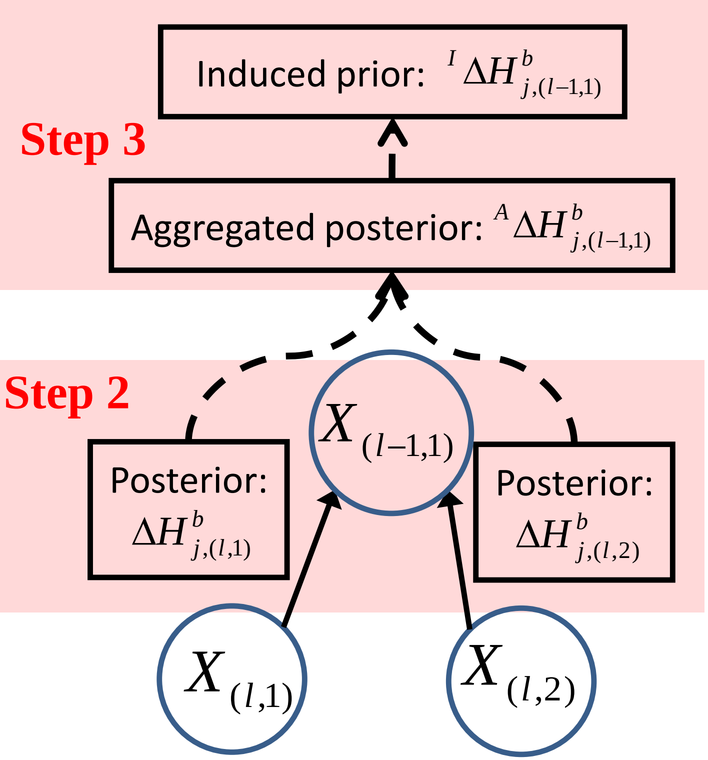
Aggregated posterior: $^{A}\Delta H^{b}_{j,(l-1,1)} = g(\Delta H^{b}_{j,(l,\kappa)}),\ \kappa = 1,2$

- **Step 3** - Approximate the induced prior:

Induced prior: $^{I}\Delta H^{b}_{j,(l-1,1)} \leftarrow {}^{A}\Delta H^{b}_{j,(l-1,1)}$

$$^{I}\Delta H^{b}_{j,(l-1,1)} \sim Gamma(^{I}\eta_{j,(l-1,1)}, {}^{I}\lambda_{j,(l-1,1)})$$

(Validate by K-S goodness fitness test)



Aggregation Procedure: Step 4

- **Step 4** – Combine the native prior and the induced prior:

Combined prior: $^{C}\Delta H^{b}_{j,(l-1,1)} \sim Gamma(^{C}\eta_{j,(l-1,1)}, {}^{C}\lambda_{j,(l-1,1)})$

$$^{C}\eta_{j,(l-1,1)} = w\,^{I}\eta_{j,(l-1,1)} + (1-w)\,^{N}\eta_{j,(l-1,1)}$$

Weighting factor: $0 \le w \le 1$

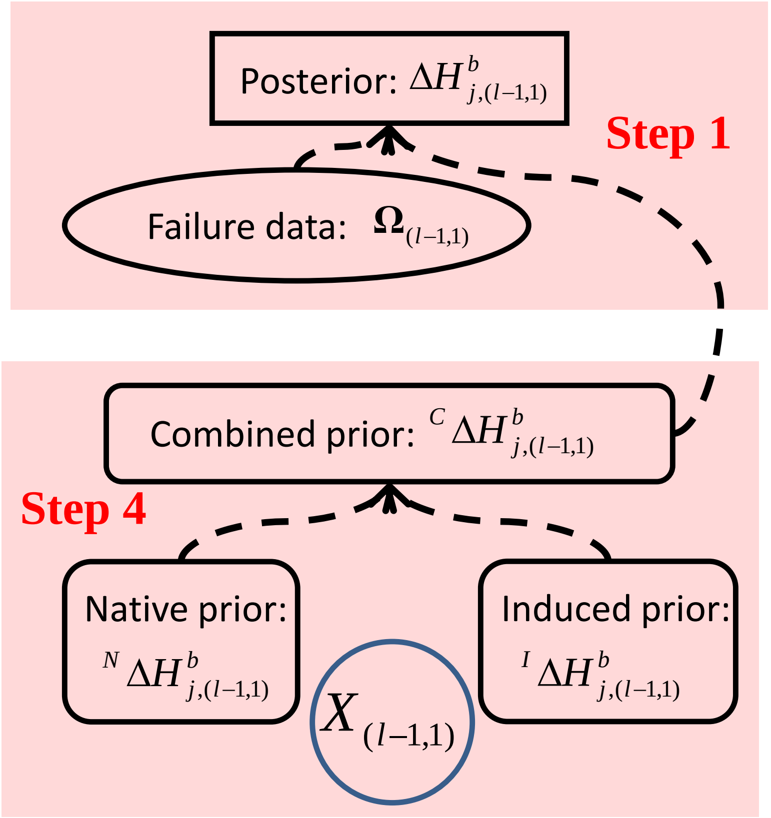
$$^{C}\lambda_{j,(l-1,1)} = w\,^{I}\lambda_{j,(l-1,1)} + (1-w)\,^{N}\lambda_{j,(l-1,1)}$$

w **: balance native prior and induced prior**

- **Step 1** – Compute the posterior (higher-level element):

Similar Bayesian inference

$$^{C}\Delta H^{b}_{j,(l-1,1)} \sim Gamma(^{C}\eta_{j,(l-1,1)}, {}^{C}\lambda_{j,(l-1,1)})$$



Posterior: $\Delta H^{b}_{j,(l-1,1)}$

Step 1

Failure data: $\mathbf{\Omega}_{(l-1,1)}$

Combined prior: $^{C}\Delta H^{b}_{j,(l-1,1)}$

Step 4

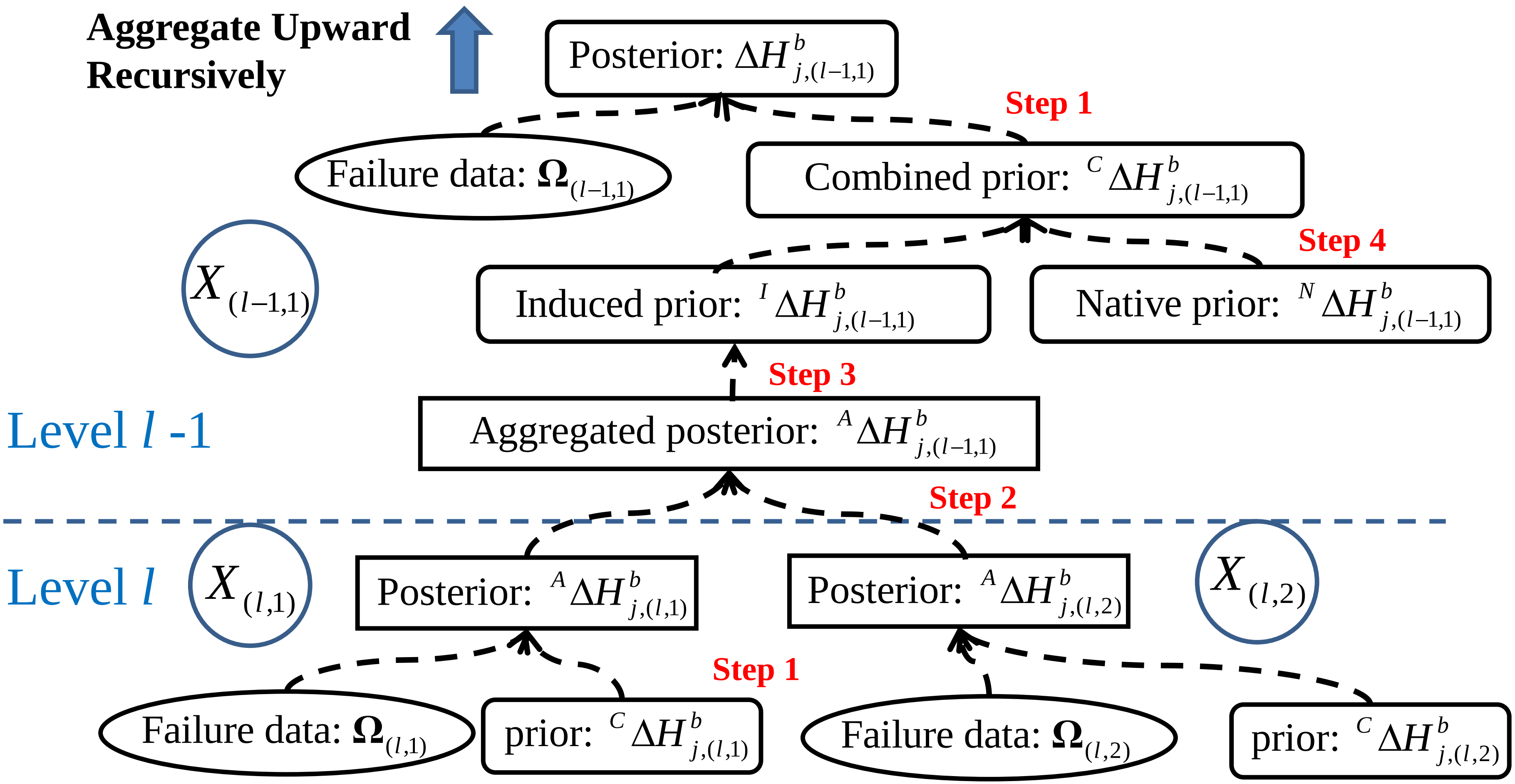
Native prior: $^{N}\Delta H^{b}_{j,(l-1,1)}$

$X_{(l-1,1)}$

Induced prior: $^{I}\Delta H^{b}_{j,(l-1,1)}$

Information Aggregation: Procedure Review

- **Recursive**
- **Flexible**
- **Generic**



Aggregate Upward Recursively

Posterior: $\Delta H^{b}_{j,(l-1,1)}$

Step 1

Failure data: $\mathbf{\Omega}_{(l-1,1)}$

Combined prior: ${}^{C}\Delta H^{b}_{j,(l-1,1)}$

Step 4

$X_{(l-1,1)}$

Induced prior: ${}^{I}\Delta H^{b}_{j,(l-1,1)}$

Native prior: ${}^{N}\Delta H^{b}_{j,(l-1,1)}$

Step 3

Level l -1

Aggregated posterior: ${}^{A}\Delta H^{b}_{j,(l-1,1)}$

Step 2

Level l

$X_{(l,1)}$

Posterior: ${}^{A}\Delta H^{b}_{j,(l,1)}$

Posterior: ${}^{A}\Delta H^{b}_{j,(l,2)}$

$X_{(l,2)}$

Step 1

Failure data: $\mathbf{\Omega}_{(l,1)}$

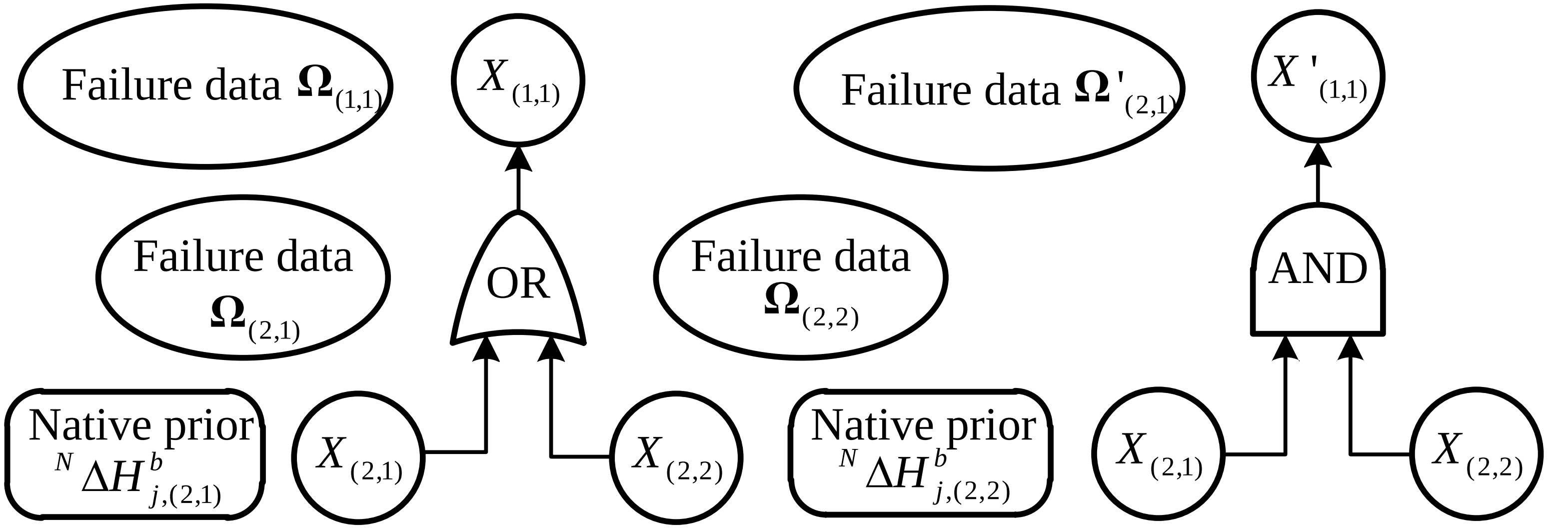
prior: ${}^{C}\Delta H^{b}_{j,(l,1)}$

Failure data: $\mathbf{\Omega}_{(l,2)}$

prior: ${}^{C}\Delta H^{b}_{j,(l,2)}$

Numerical Case Study

- A two-level hierarchical system with 3 elements
- One covariate u is considered with binary values: 0/1
- Test data are simulated with 30 intervals



Series system

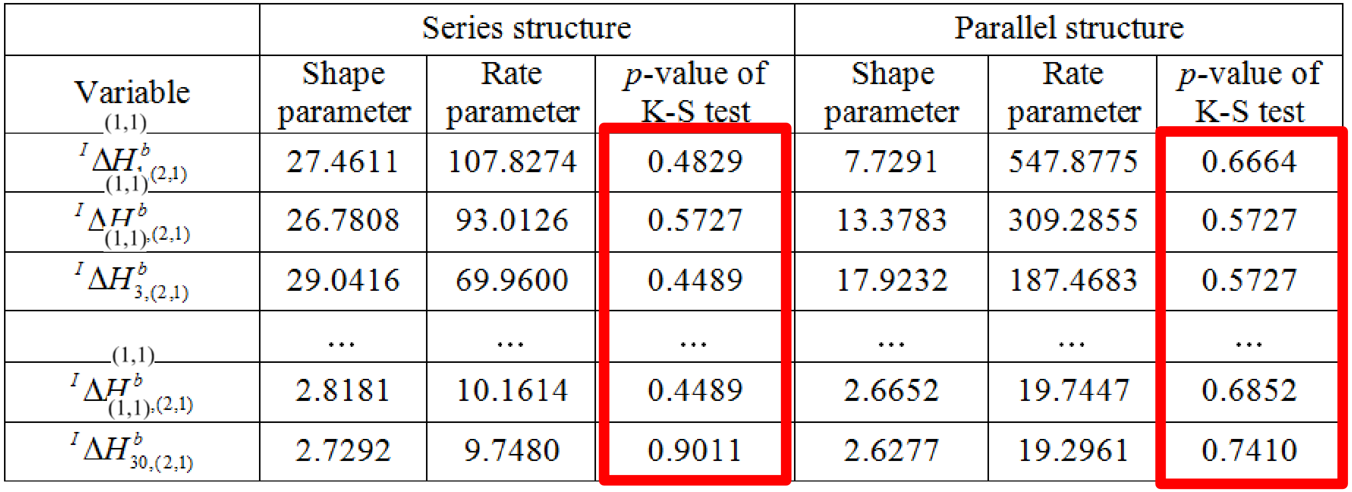
Parallel system

Information Aggregation

- Steps 1-2: Compute and aggregate the posteriors of components:

$$^{A}\Delta H^{b}_{j,(l-1,k)} = g(\Delta H^{b}_{j,(l,\kappa)}),\ \kappa \in \mathbf{Q}_{(l-1,k)}$$

- Step 3: Approximate the aggregated posteriors into the induced priors:



	Series structure			Parallel structure		
Variable	Shape parameter	Rate parameter	*p*-value of K-S test	Shape parameter	Rate parameter	*p*-value of K-S test
$^{I}\Delta H^{b}_{(1,1),(2,1)}$	27.4611	107.8274	0.4829	7.7291	547.8775	0.6664
$^{I}\Delta H^{b}_{(1,1),(2,1)}$	26.7808	93.0126	0.5727	13.3783	309.2855	0.5727
$^{I}\Delta H^{b}_{3,(2,1)}$	29.0416	69.9600	0.4489	17.9232	187.4683	0.5727
	…	…	…	…	…	…
$^{I}\Delta H^{b}_{(1,1),(2,1)}$	2.8181	10.1614	0.4489	2.6652	19.7447	0.6852
$^{I}\Delta H^{b}_{30,(2,1)}$	2.7292	9.7480	0.9011	2.6277	19.2961	0.7410

Information Aggregation (Cont’d)

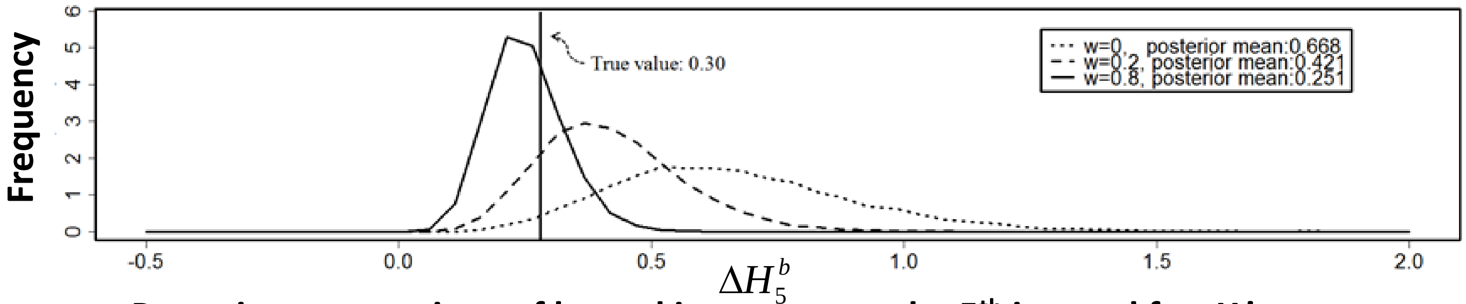
- Step 4: Combine the induced priors and the native priors

w=0, 0.2, 0.8 Different effects of information aggregation

- Step 1: Compute posteriors for the system

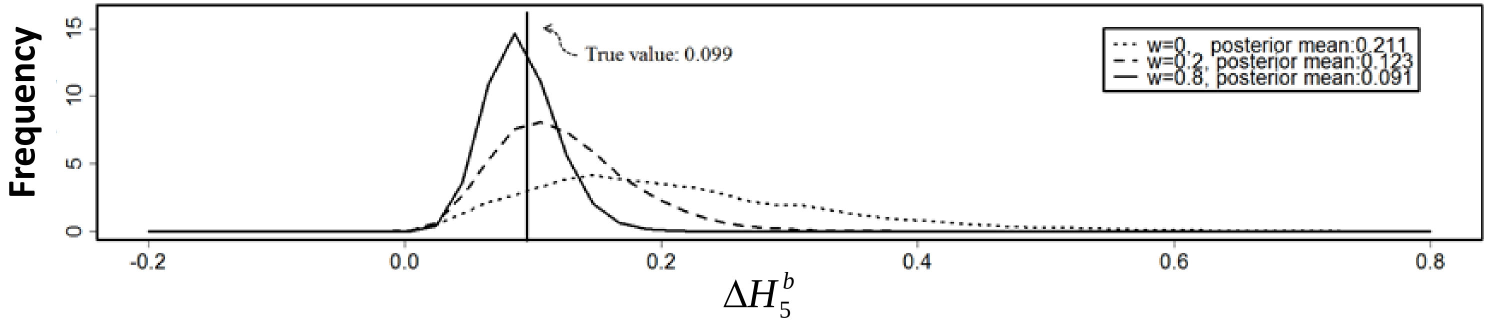
Posteriors comparison of hazard increment at the 5th interval for $X_{(1,1)}$

Series:

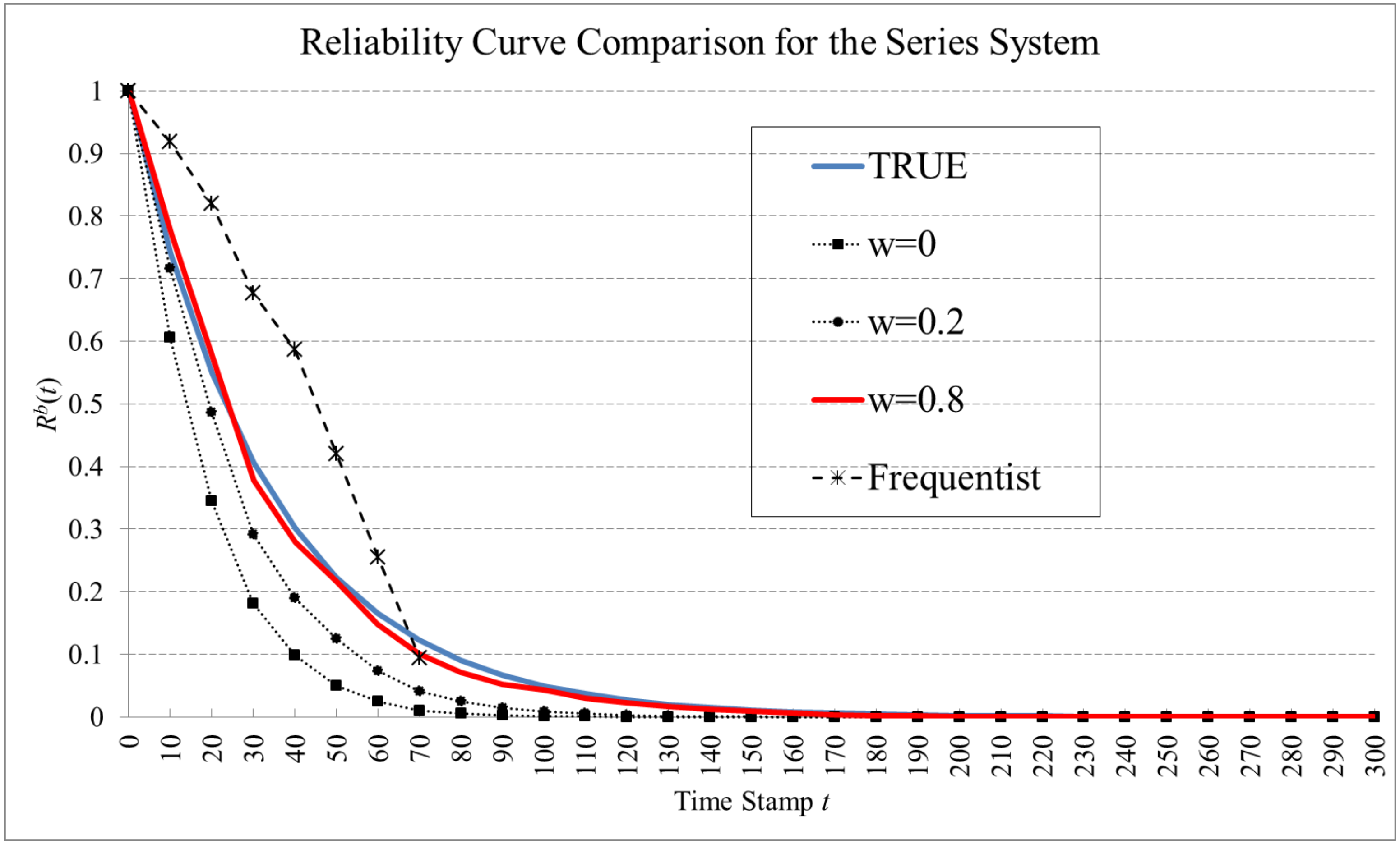


Posteriors comparison of hazard increment at the 5th interval for $X'_{(1,1)}$

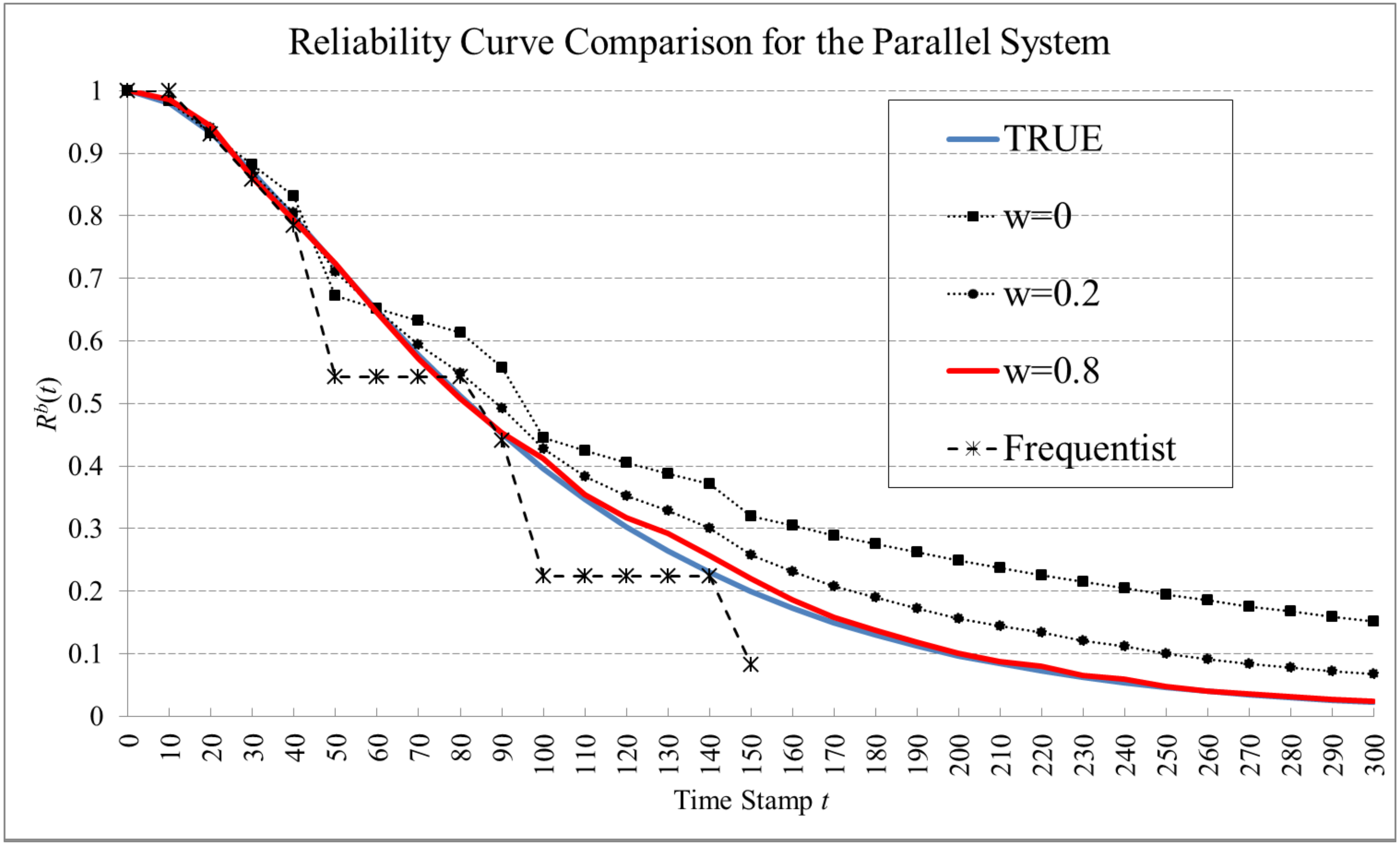
Parallel:



Series System Reliability Curve Comparisons



Parallel System Reliability Curve Comparisons



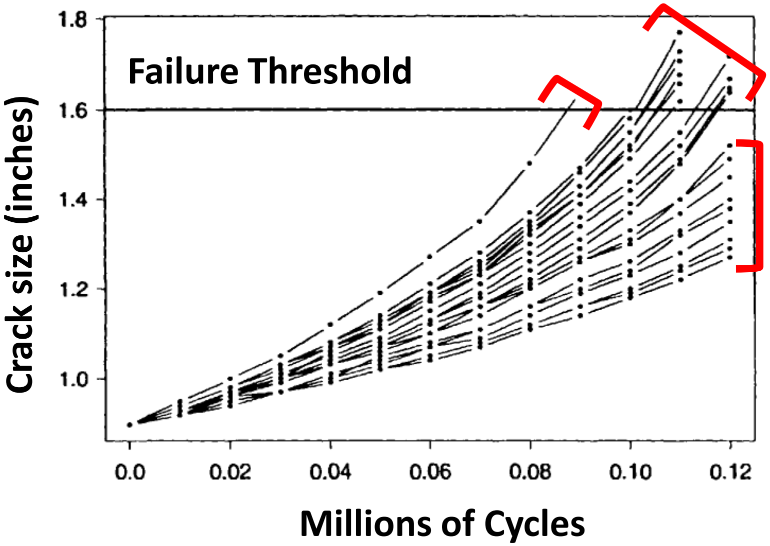
Part II - Heterogeneous Data Quantification:

Bayesian Modeling and Learning of Heterogeneous Time-to-Event Data with an Unknown Number of Sub-populations

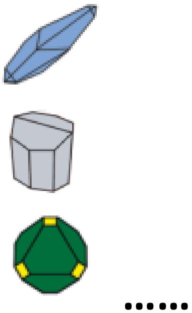
Vision

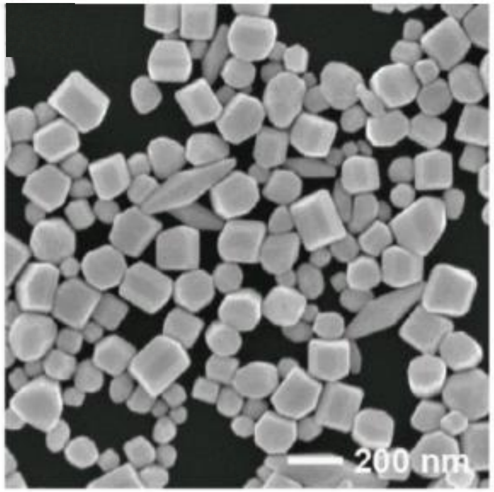
Health Care Utilization Data[19]

Alloy Fatigue Crack Size Data[20]



Nanocrystals Growth Data[21]





Heterogeneous Populations:

Heterogeneous Data

Quantification

How many? How to model?

Focus: Time-to-Event Data

- Time-to-event (TTE) data is important

TTE: Time to occurrence of an **event** of interest

Event

Contract renew

Surgery completion

Machine breakdown

Product recall

Occurrence of a disease

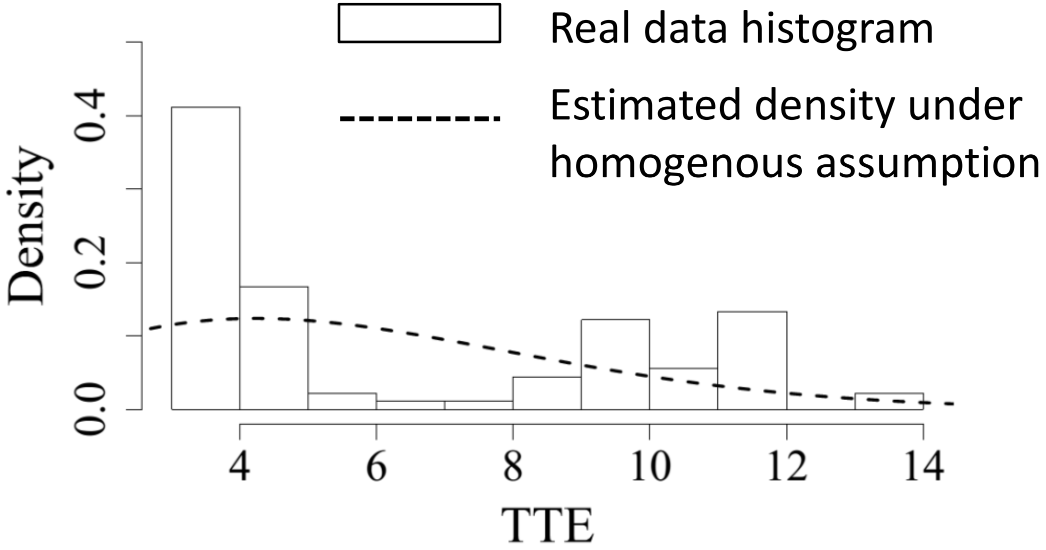
…

GENERAL
&
CRITICAL

TTE Heterogeneity

- TTE: assembly time

Intelligent Robotic Assembly System[22]



Histogram of data at the **SAME** process setting

- **~~Homogenous assumption~~**
- **Reason**: heterogeneous products quality, etc.

TTE Heterogeneity (Cont'd)

- Reliability examples
- **Semiconductor industry**[23]: infant mortality failures

  Reason: manufacturing defects, assembly errors, etc.
- **Automobile industry**[24]: early failures

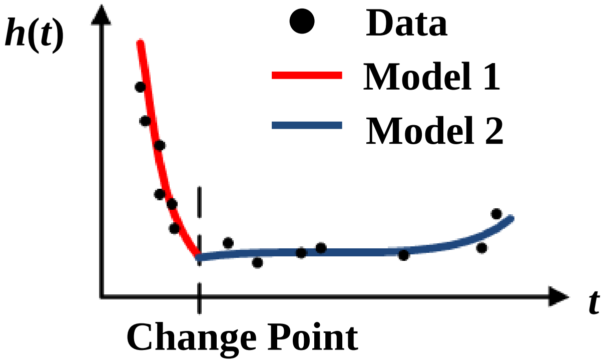
  Reason: material quality, unverified design changes, etc.
- **Industry with evolving technology**[25]:heterogeneity especially critical

  Reason: immature technology

Q: How to model TTE heterogeneity?

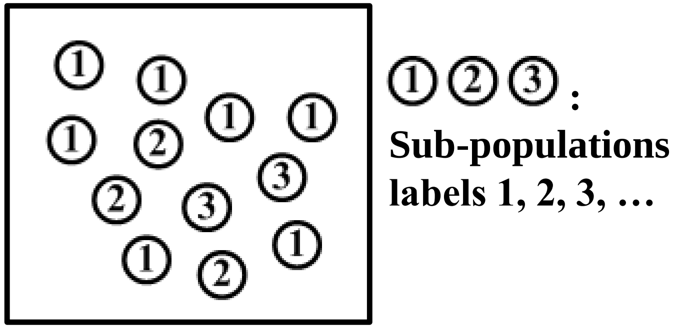
Heterogeneity Modeling of TTE

- **Change point model**[26-28]:



Limitation: different domains

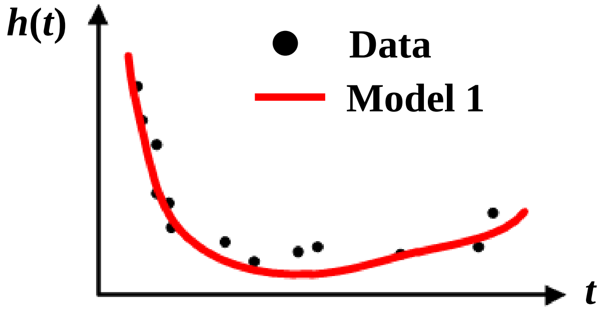
- **Frailty model**[29,30] :



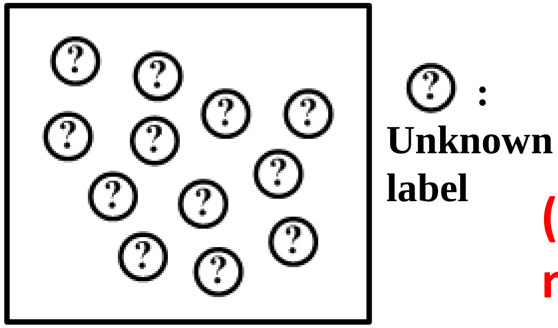
Limitation: known membership

- **Mixture model**[31,32]:

Scope



(1) One model under entire domain



(2) Unknown membership

(3) Meaningful interpretation;
(4) Feedback information.

Mixture Model: Gaps and Solutions

Existing Method	Limitation	Advantage	Solution
Known the number of sub-populations (***m***)[33,34]	subjective	**Unknown** ***m***, learned from data	objective
Model estimation + model selection (e.g., LRT, AIC)[35,36]	**Two-step**	Bayesian formulation	**Joint** model estimation and selection
Mixtures of **distributions**[32,37,38,41]	w/o covariates	Mixtures of **regressions**	w/ covariates
Conjugate prior[39,40]	Restrictive	**Non-conjugate** prior	Generic

Expected Features of the Proposed Work

- Assuming an **unknown** # of sub-populations
- Considering influence of possible **covariates**
- Achieving **joint** model estimation and model selection
- Comprehensive treatment of **non-conjugate** priors

Mixture Model: Known *m*

- j^{th} homogenous sub-population:

$$h_j(t\,|\,\mathbf{x}) = h_j^{\mathrm{b}}(t)\exp(\boldsymbol{\beta}_j^T\mathbf{x})$$

Covariates

Hazard function

Covariate coefficients

TTE

Baseline Hazard function

Benefits: (1) Covariates; (2) Flexible; (3) $h_j(t) \overset{\textbf{Unique}}{\leftrightarrow} f_j(t)$

- The overall heterogeneous population:

What if unknown

***m* known**

$$g(t\,|\,\boldsymbol{\Theta}^m,\mathbf{x}) = \sum_{j=1}^{\boxed{m}} w_j f_j(t\,|\,\boldsymbol{\theta}_j,\mathbf{x})$$

Population pdf

Sub-population pdf

All unknowns

Sub-population proportion

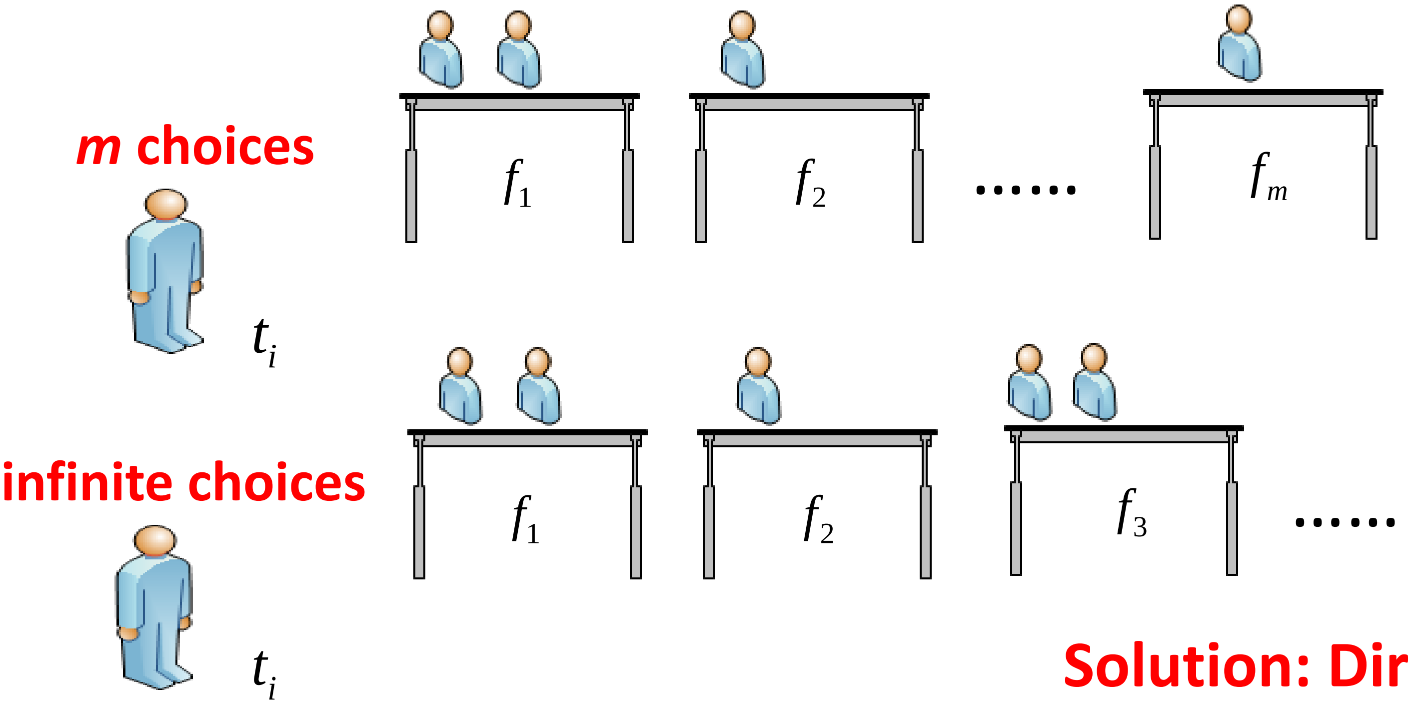
Sub-population unknowns

Mixture Model: Unknown *m*

- Finite mixture model:

***m* known**

$$g(t \mid \boldsymbol{\Theta}^m, \mathbf{x}) = \sum_{j=1}^{\boxed{m}} w_j f_j(t \mid \boldsymbol{\theta}_j, \mathbf{x})$$



Solution: Dirichlet Process

Mixture Model: Unknown *m* (Cont’d)

- Bayesian hierarchical formulation:

$$
\begin{aligned}
&t \mid \mathbf{x}, \boldsymbol{\theta} \sim f(\cdot \mid \mathbf{x}, \boldsymbol{\theta}),\\
&\boldsymbol{\theta} \mid P \sim P\\
&P \mid \alpha, P_0 \sim \mathbf{DP}(\alpha P_0(\cdot))
\end{aligned}
\quad \Longrightarrow \quad
g(t \mid \boldsymbol{\Theta}, \mathbf{x}) = \sum_{j=1}^{\boxed{\infty}} w_j f_j(t \mid \boldsymbol{\theta}_j, \mathbf{x})
$$

A random distribution

Dirichlet process

Positive scalar

Base distribution

finite mixture: $\sum_{j=1}^{m}$

$$\Downarrow$$

infinite mixture: $\sum_{j=1}^{\infty}$

New formulation:

(1) **no restriction** on *m*

(2) *m* learned **objectively**

(3) **Joint** model estimation and model selection

Estimation Challenges

Data: $\mathbf{D}=\{t_i,\Delta_i,\mathbf{x}_i\}_{i=1}^{n}$

Right-censored indicator

Unknowns: $\boldsymbol{\Theta}=\{w_j,\boldsymbol{\beta}_j,k_j,\eta_j\}_{j=1}^{\infty}$

Weibull baseline shape scale

Joint posterior:

$$\pi(\boldsymbol{\Theta}\,|\,\mathbf{D}) \propto \prod_{i=1}^{n}\left(\sum_{j=1}^{\infty} w_j f_j(t_i\,|\,\overset{\textcircled{1}}{\boldsymbol{\beta}_j,k_j,\eta_j},\mathbf{x}_i)\right)^{\Delta i} \cdot\left(\sum_{j=1}^{\overset{\textcircled{3}}{\infty}} w_j R_j(t_i\,|\,\boldsymbol{\beta}_j,k_j,\eta_j,\mathbf{x}_i)\right)^{1-\Delta i} \cdot \overset{\textcircled{2}}{\pi(\boldsymbol{\Theta})}$$

Challenges:

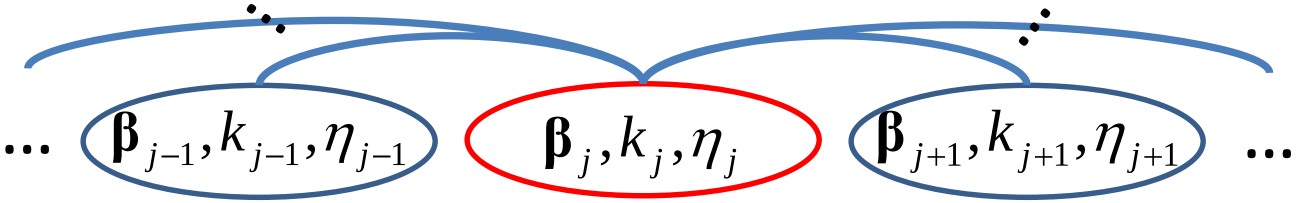
① **High** dependency ➡ Slow/failed convergence

② **Non-conjugate** prior ➡ Sampling difficulty

③ **Infinite** # of unknowns ➡ Computationally formidable

Estimation Solutions

① High dependency:



$\mathbf{Z} = \{z_i\}_{i=1}^{n}$

labels for t_i’s

② Non-conjugate prior:

$\pi(\eta_j \mid \cdot)$ → M-H → **Reparameterization**: $\lambda_j = \eta_j^{-k_j}$

$\pi(\lambda_j \mid \cdot)$: Gamma

$\pi(k_j \mid \cdot)$ → M-H → **ARS**, condition **provided**

$\pi(\boldsymbol{\beta}_j \mid \cdot)$ → M-H → **ARS**, condition **provided**

Metropolis-Hasting (M-H)[41]:

- Pros: General purpose
- Cons: Tuning problem, samples auto-correlated

Adaptive Rejection Sampling (ARS)[42]:

- Condition-based
- No turning, samples independent

③ Infinite # of unknowns: slice-sampling techniques[40]: *j*=1,2,…,J^*, where J^* is **finite**

Realized Features of the Proposed Work

- **Unknown** # of sub-populations: **Dirichlet process**
- **Covariates**: **hazard regression**
- **Joint** model estimation & selection: **Bayesian model**
- **Non-conjugate** priors: **a series of sampling techniques**

Numerical Case Study: Effectiveness

- **Simulation setup**
- 2-mixture of Weibull regression
- Single covariate X~Unif(0,5)
- Right-censored time 1.0e+5

2 sub-populations

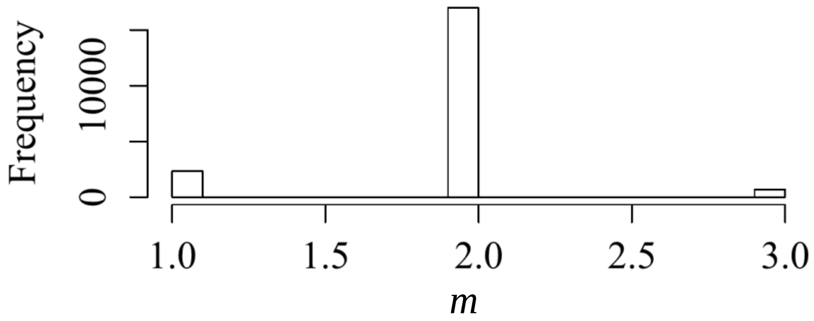
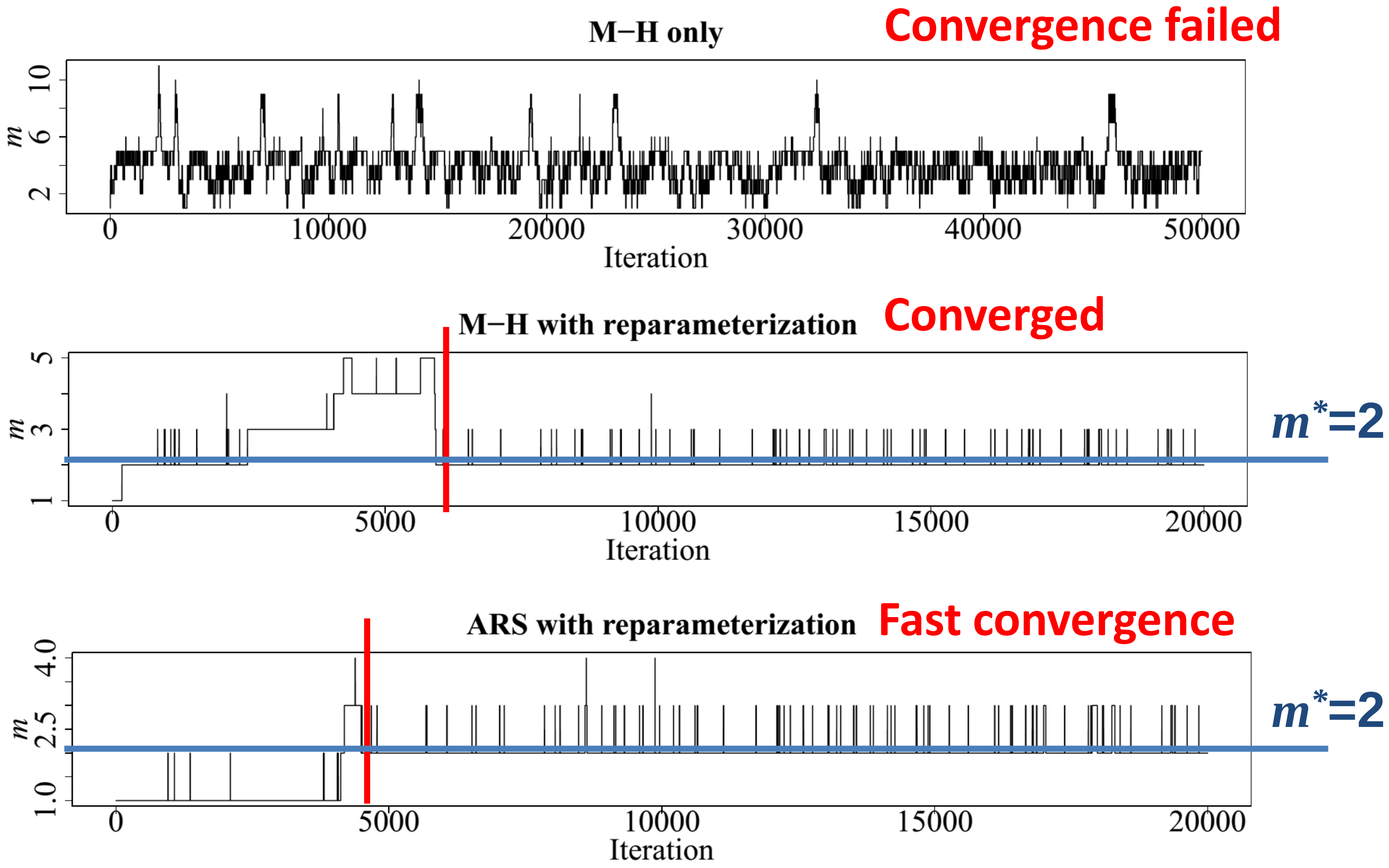


Figure 1. Model selection results

Table 1. Model estimation results

	Sub-population 1				Sub-population 2			
Parameter	p_1	k_1	η_1	β_1	p_2	k_2	η_2	β_2
True value	0.3	0.7	2.0e+3	1	0.7	3	8.0e+4	0.5
Estimate	**0.28**	**0.66**	**1.95e+3**	**0.94**	**0.66**	**2.97**	**8.14e+4**	**0.51**

Efficiency



Convergence failed

Converged

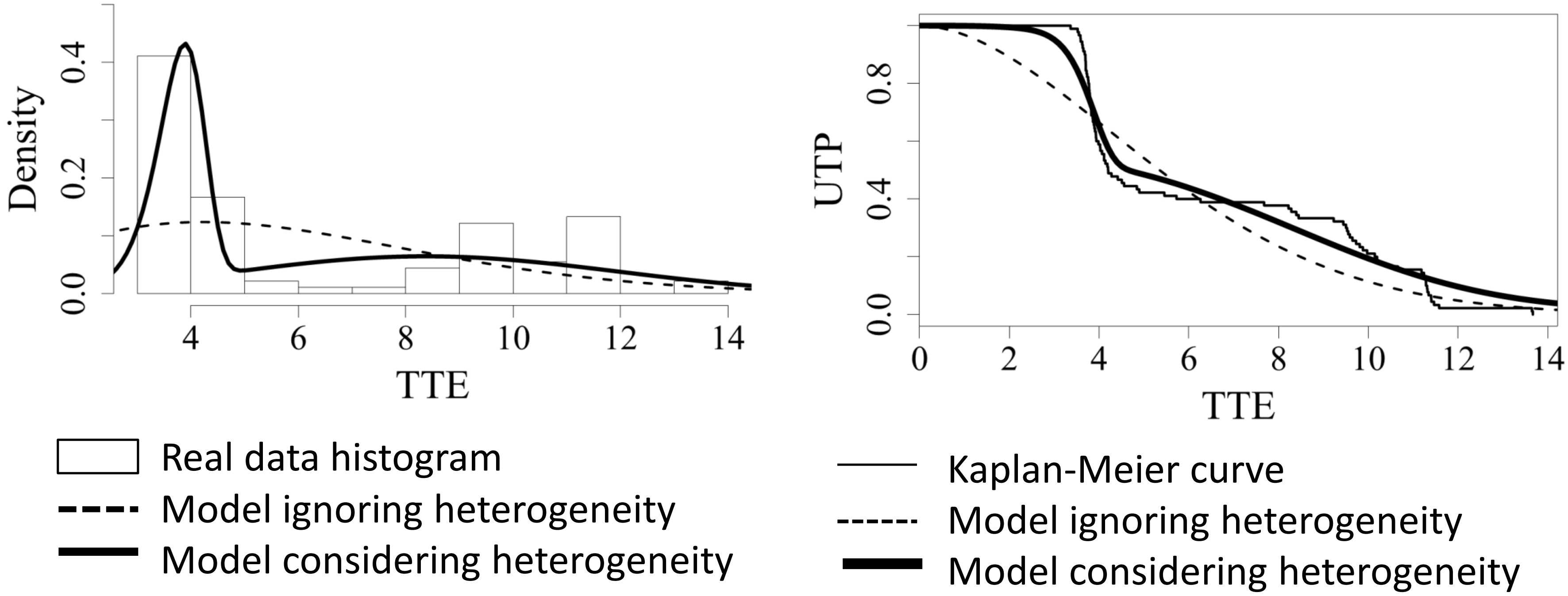
$m^*=2$

Fast convergence

$m^*=2$

Real data analysis

- Assembly time data



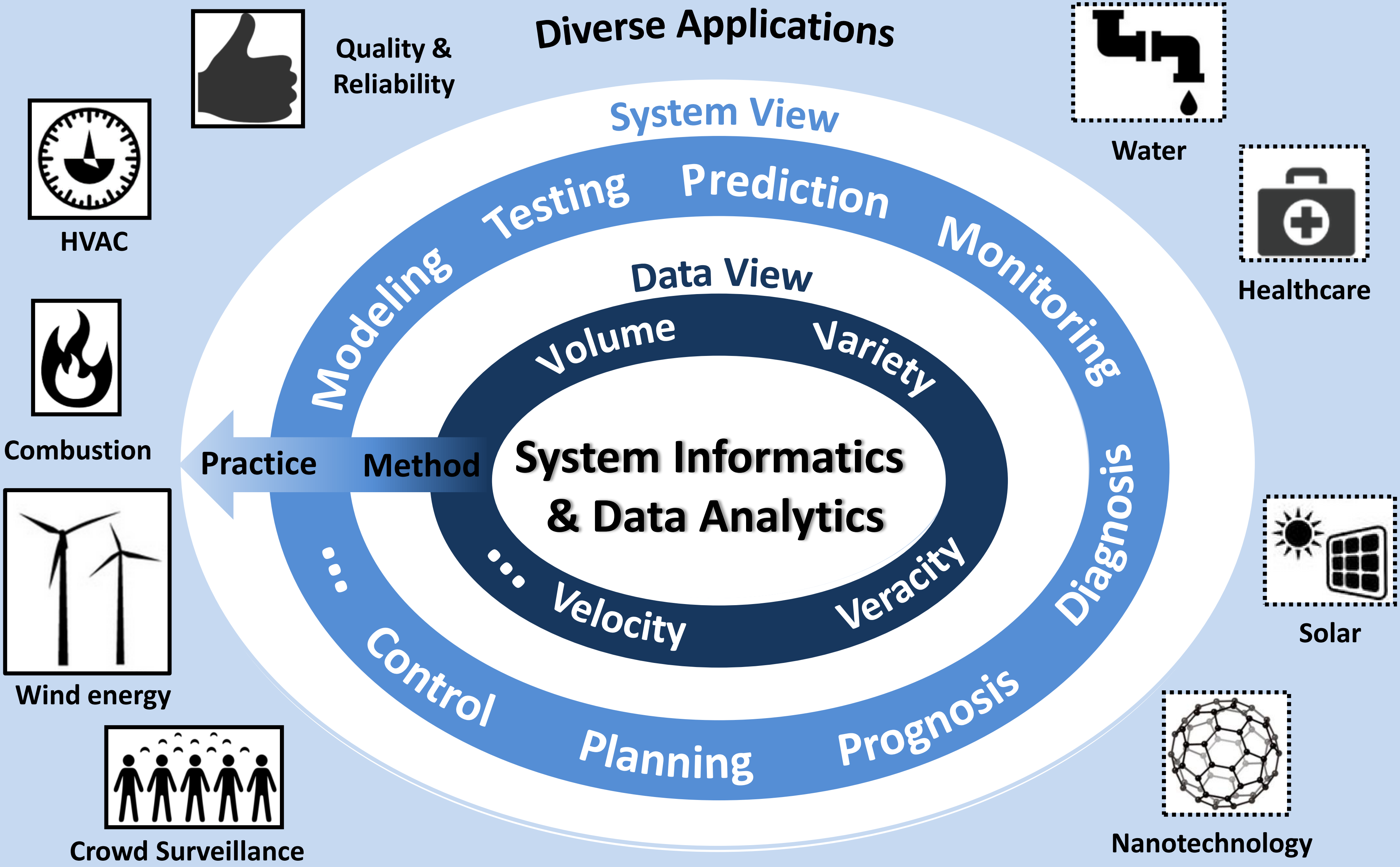
UTP: Unfinished Task Probability

(a) Estimated densities comparison

(b) UTP curves comparison

Figure 4. Comparisons of models w/ and w/o considering heterogeneity

Summary



Diverse Applications

Quality & Reliability

HVAC

Combustion

Wind energy

Crowd Surveillance

Water

Healthcare

Solar

Nanotechnology

System View

Testing

Prediction

Monitoring

Diagnosis

Prognosis

Planning

Control

…

Modeling

Data View

Volume

Variety

Veracity

Velocity

…

System Informatics & Data Analytics

Practice

Method

Thanks ☺

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