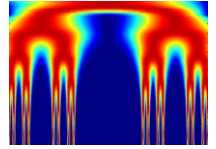


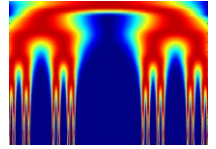
Multiscale Analysis of Recurrence Patterns

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Complex Systems Monitoring, Modeling and Analysis Lab
University of South Florida



- ❑ Problem Statement
- ❑ Research Background
 - Wavelet transform
 - Recurrence analysis
- ❑ Research Methodology
 - Multiscale recurrence analysis
 - Feature selection
 - Randomized classification experiments
- ❑ Experimental Designs & Results
- ❑ Conclusions



❑ Challenges:

- “Big data”
- Data complexity: **nonlinear** and **nonstationary**

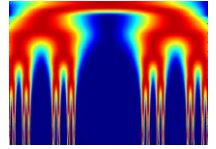
❑ State of the Art:

- Conventional frequency-domain and linear system approaches
- Nonlinear stochastic dynamics under highly nonstationary conditions
- Nonlinear dynamic methods – **computational expensive**

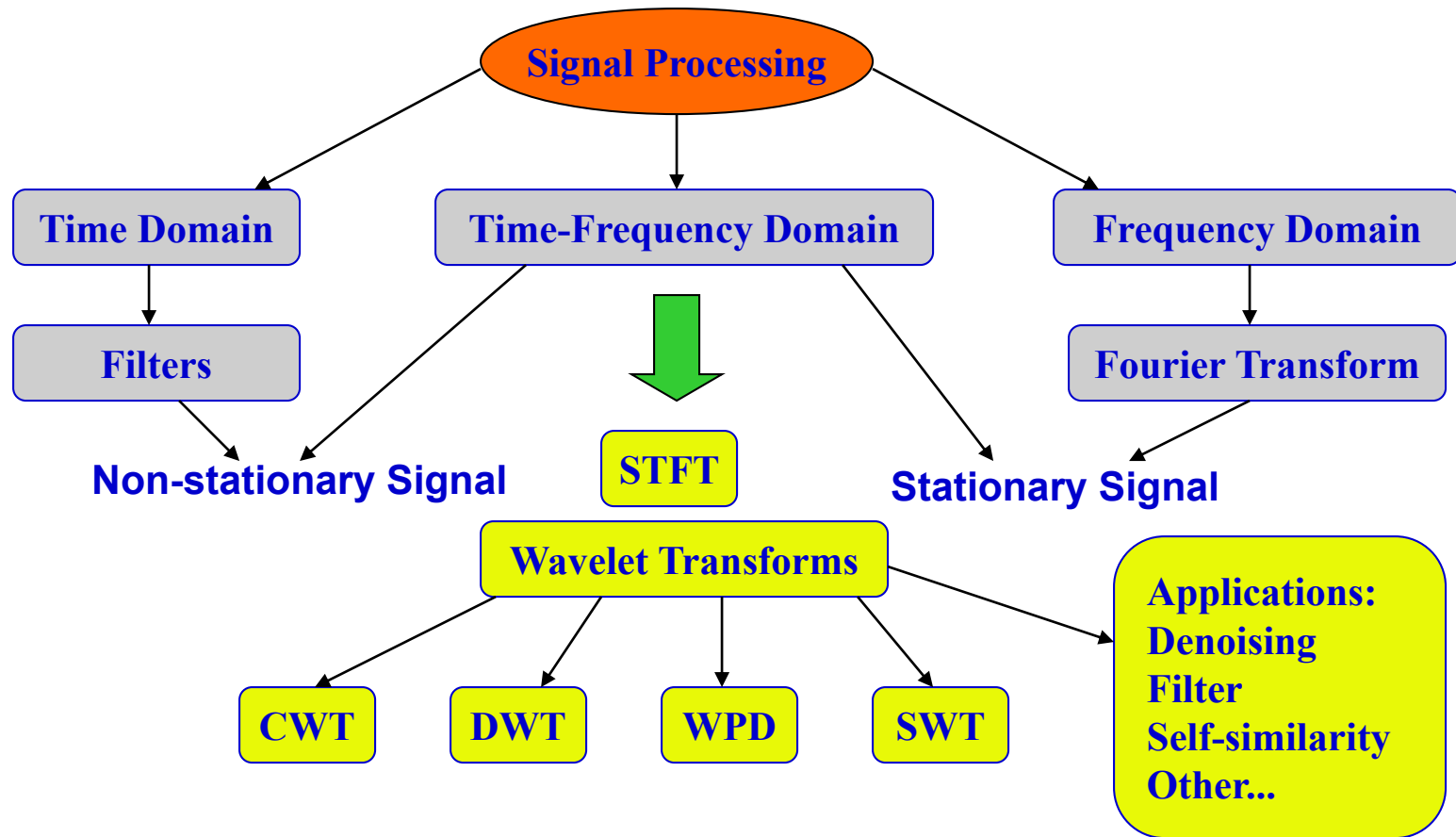
❑ Research Objectives:

- Address the issues of nonlinearity, nonstationarity and large datasets
- Extended and integrated into other nonlinear dynamic approaches
- Disease-altered nonlinear dynamics

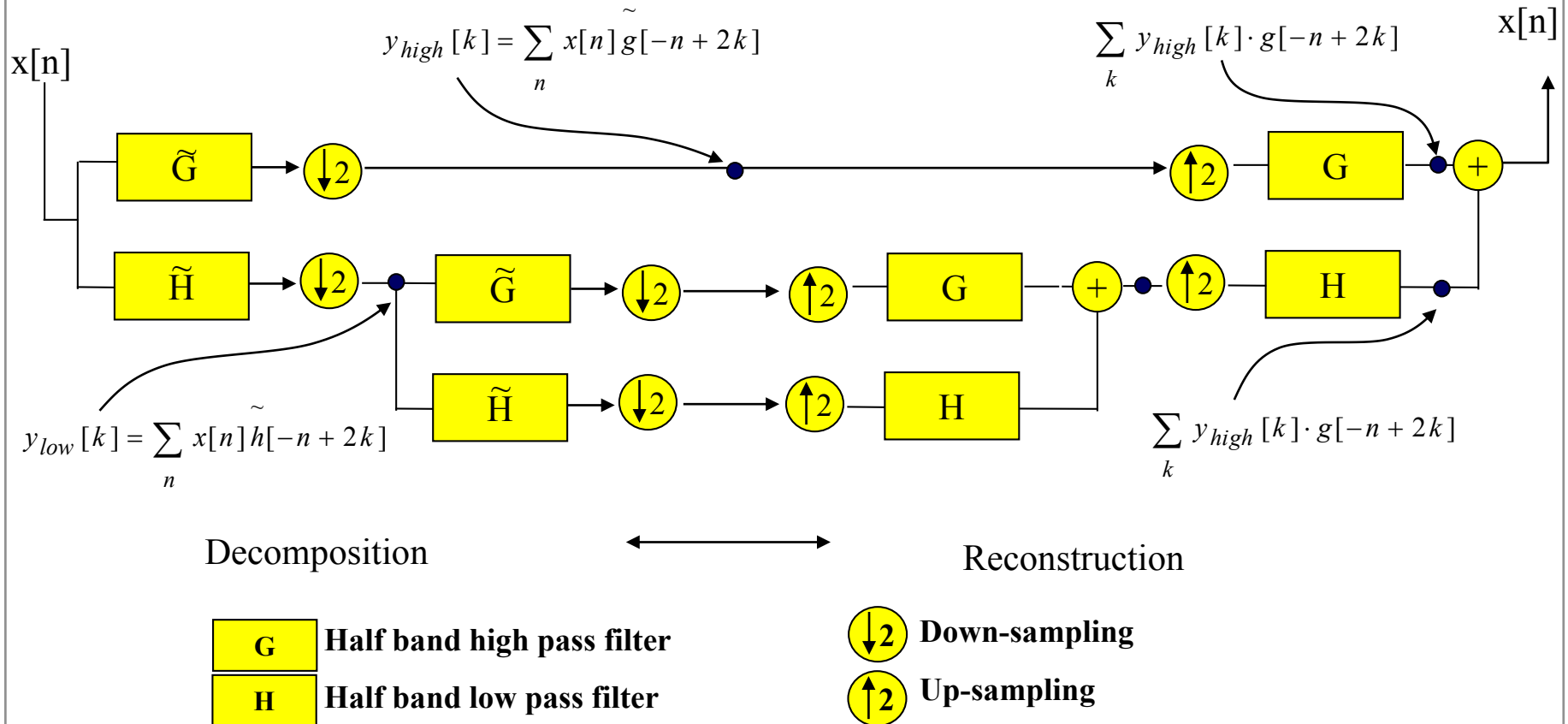
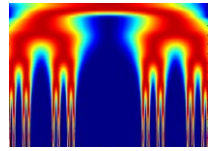
Signal Processing



- ☐ Time info difficult to interpret in frequency domain
- ☐ Frequency info difficult to interpret in time domain
- ☐ Perfect time info in time domain, perfect freq. info in freq. domain

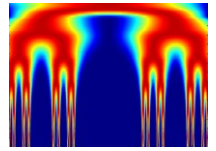


Discrete Wavelet Transform



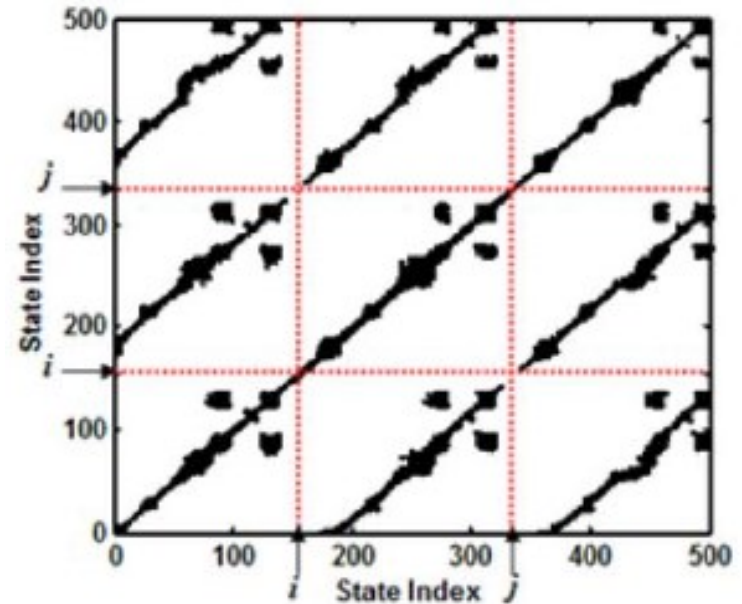
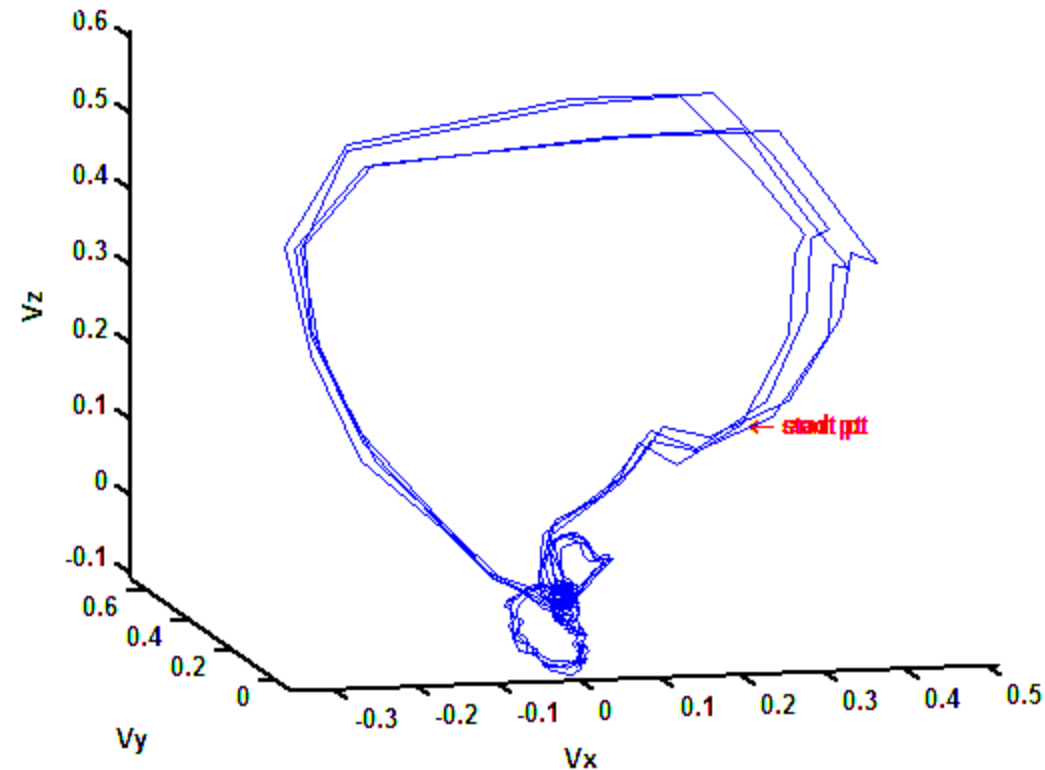
2-level DWT decomposition & wavelet filter bank

Recurrence Analysis

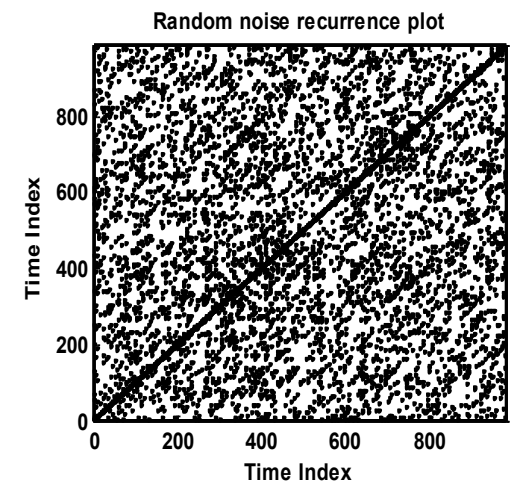
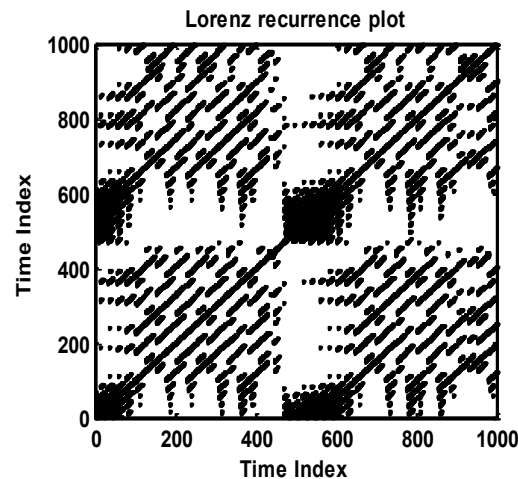
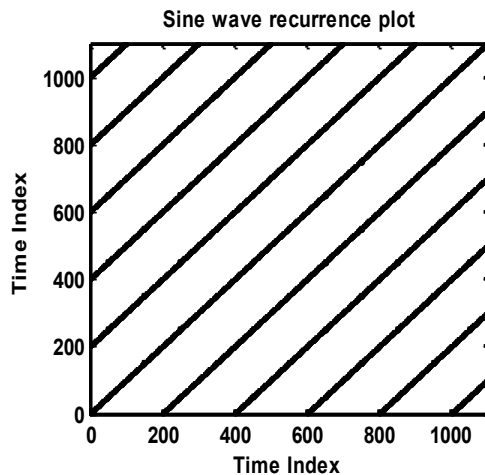
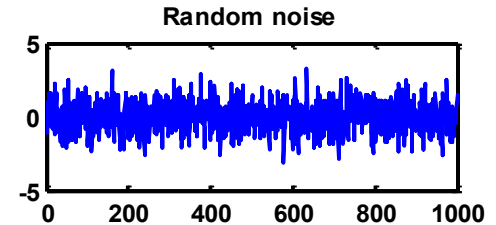
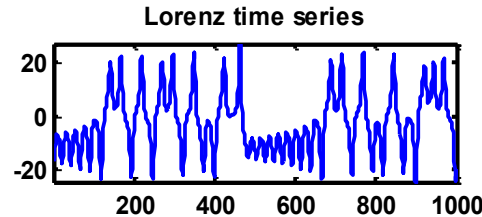
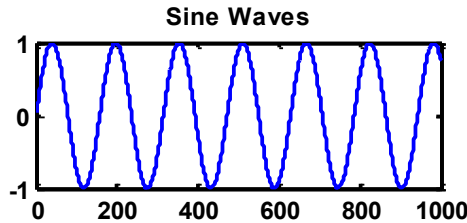
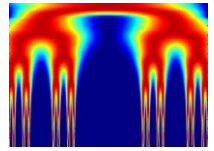


□ Recurrence patterns of the dynamical systems

➤ Recurrence plot: $R(i, j) = \Theta(\varepsilon - \|x(i) - x(j)\|)$



Structures in Recurrence Plots



Linear System

Nonlinear System

Randomized System

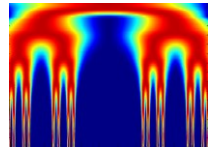
❑ Small-scale structures

- single dots, diagonal and vertical lines

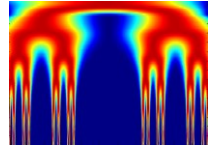
❑ Large-scale structures

- homogenous, periodic and disrupted visualization

Recurrence Features

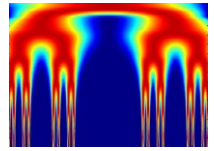


- ❑ Quantifying the *topological features of Recurrence Plots*
- ❑ **Statistical features** to quantify certain recurrence patterns from Threshold Recurrence Plot (Kantz, Marwan, and Kurths et al.):
 - Recurrence rate (%REC)
 - Determinism (%DET)
 - Linemax (LMAX)
 - Entropy (ENT)
 - Laminarity (%LAM)
 - Trapping time (TT)
- ❑ **Diagonal structures** (first four) and **vertical structures** (last two) in the threshold recurrence plot
- ❑ **Computational complexity:** square increase



- ❑ Problem Statement
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- ❑ Conclusions

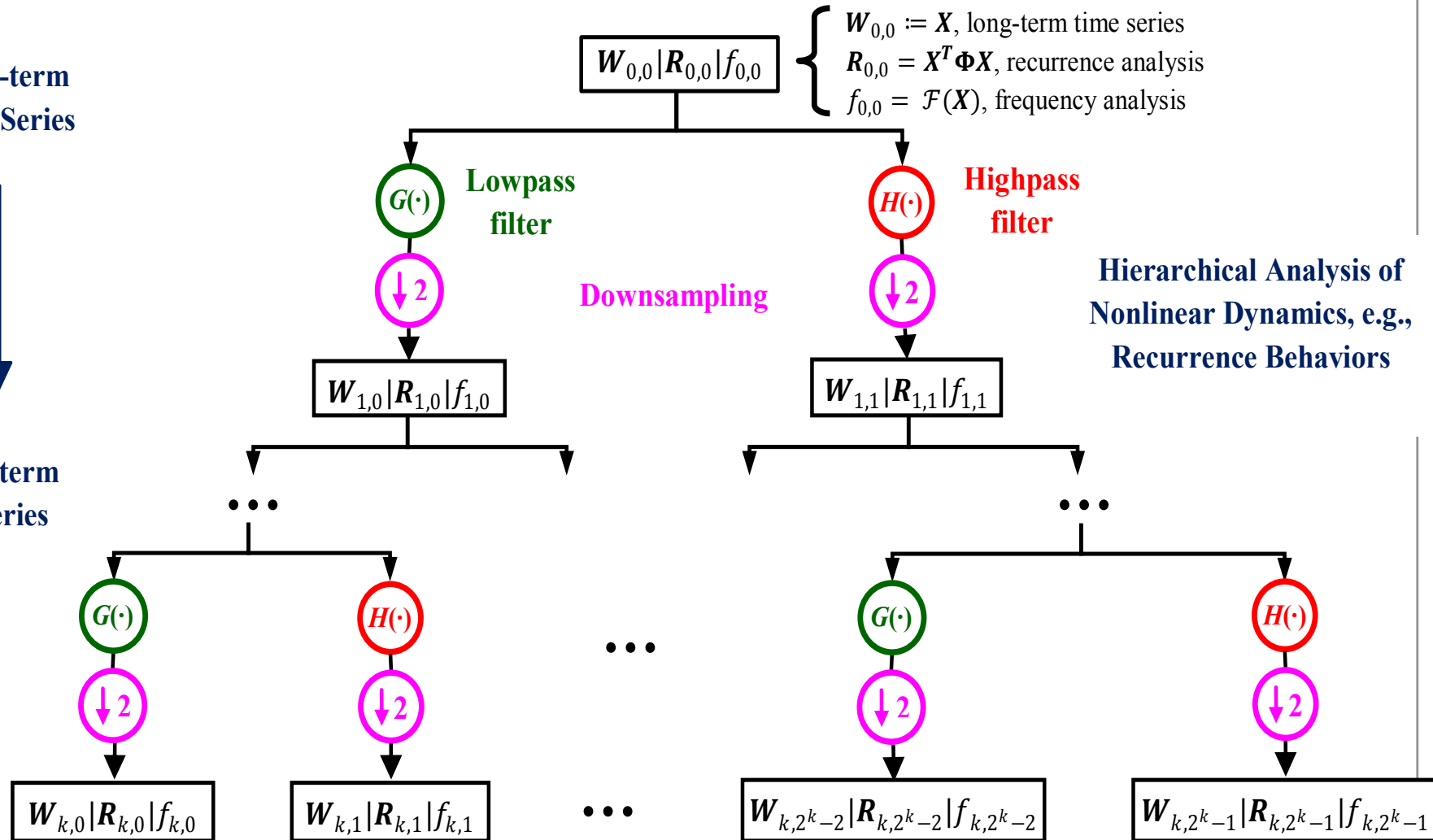
Hierarchical Nonlinear Dynamics



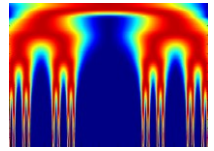
**Long-term
Time Series**



**Short-term
Subseries**



Completeness of MRA



- ❑ Given a time series $\mathbf{X} = \{x_1, x_2, \dots, x_N\}^T$
- ❑ Embedded state space $\mathbf{x}(i) = (x_i, x_{i+\tau}, \dots, x_{i+\tau(M-1)})$
- ❑ Recurrence distance matrix \leftarrow time series:

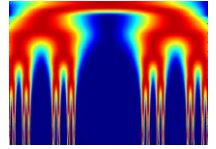
$$UR_{0,0}^2(i, j) = \|\mathbf{x}(i) - \mathbf{x}(j)\|^2 = \sum_{m=0}^{M-1} |x_{i+m\tau} - x_{j+m\tau}|^2$$

$$= \sum_{m=0}^{M-1} (\mathbf{X}^T \Phi_{i+m\tau, j+m\tau} \mathbf{X})$$

$\Phi_{i,j}$ positive semidefinite, 1 in the ii^{th} and jj^{th} elements, -1 in the ij^{th} and ji^{th} elements and 0 otherwise.

- ❑ Recurrence distance matrix \leftarrow wavelet subseries:

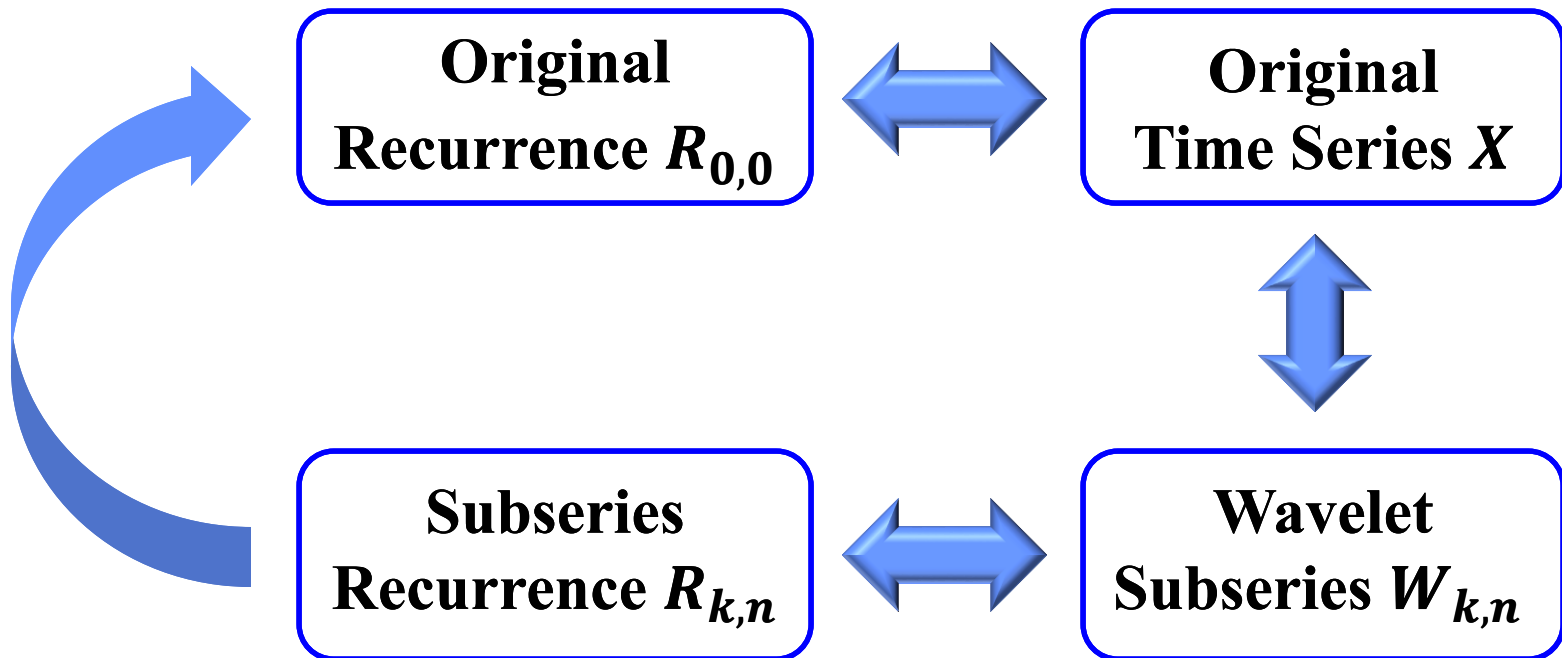
$$UR_{0,0}^2(i, j) = [\mathbf{W}_{k,2^{k-1}}^T \cdots \mathbf{W}_{k,0}^T] \mathcal{W}_k \left(\sum_{m=0}^{M-1} \Phi_{i+m\tau, j+m\tau} \right) \mathcal{W}_k^T \begin{bmatrix} \mathbf{W}_{k,2^{k-1}} \\ \vdots \\ \mathbf{W}_{k,0} \end{bmatrix}$$

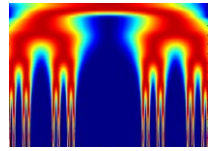


- ❑ Reconstruct time series from recurrence plot

➤ **Y. Hirata et al. (2008); M. Thiel et al. (2004)**

Recurrence Plot \longleftrightarrow Time Series





❑ Time-delay state space: $\mathbf{x}(i) = (x_i, x_{i+\tau}, \dots, x_{i+\tau(M-1)})$

❑ **Gram matrix:** $G(i, j) \equiv \mathbf{x}(i) \cdot \mathbf{x}(j)$

❑ Multidimensional scaling

$$UR^2(i, j) = [\mathbf{x}(i) - \mathbf{x}(j)] \cdot [\mathbf{x}(i) - \mathbf{x}(j)]$$

$$G(i, j) = -\frac{1}{2} \left[UR^2(i, j) - \frac{1}{N} \sum_{k=1}^N UR^2(i, k) - \frac{1}{N} \sum_{k=1}^N UR^2(k, j) + \frac{1}{N^2} \sum_{g=1}^N \sum_{h=1}^N UR^2(g, h) \right]$$

❑ Gram matrix is a square matrix: $G = U\Lambda U^T$

➤ Λ is a diagonal matrix formed from the eigenvalues of G

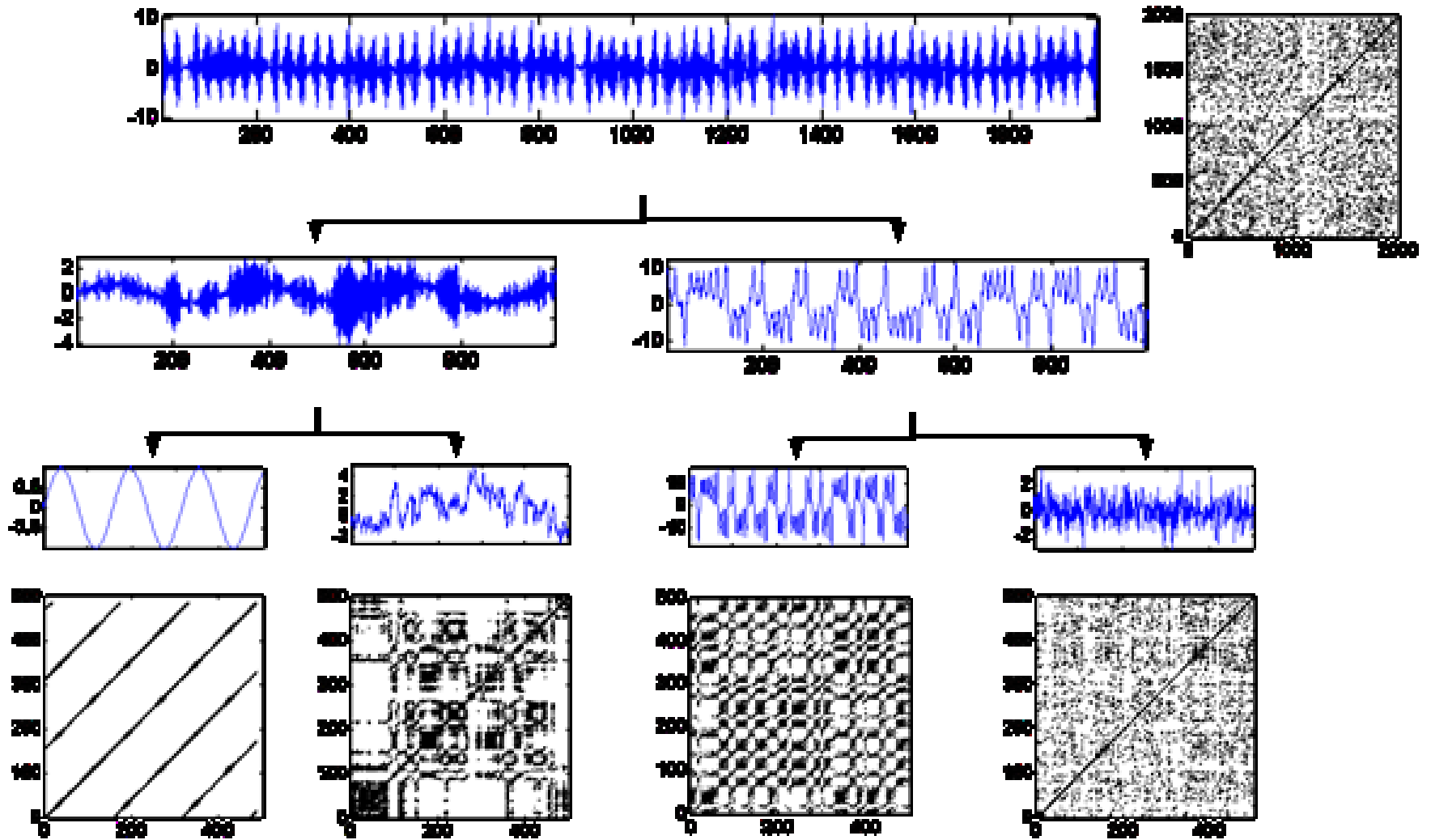
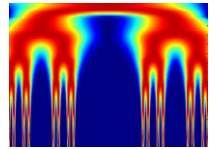
➤ U is a matrix of the corresponding eigenvectors of G

❑ Gram matrix is positive semidefinite

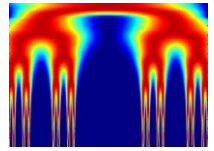
➤ $G = U\sqrt{\Lambda}\sqrt{\Lambda}U^T = U\sqrt{\Lambda}(U\sqrt{\Lambda}^T)^T = U\sqrt{\Lambda}(U\sqrt{\Lambda})^T$

❑ **ISOMETRY:** $U\sqrt{\Lambda}$ and \mathbf{x}_i

Multiscale Recurrence Analysis



Feature Selection



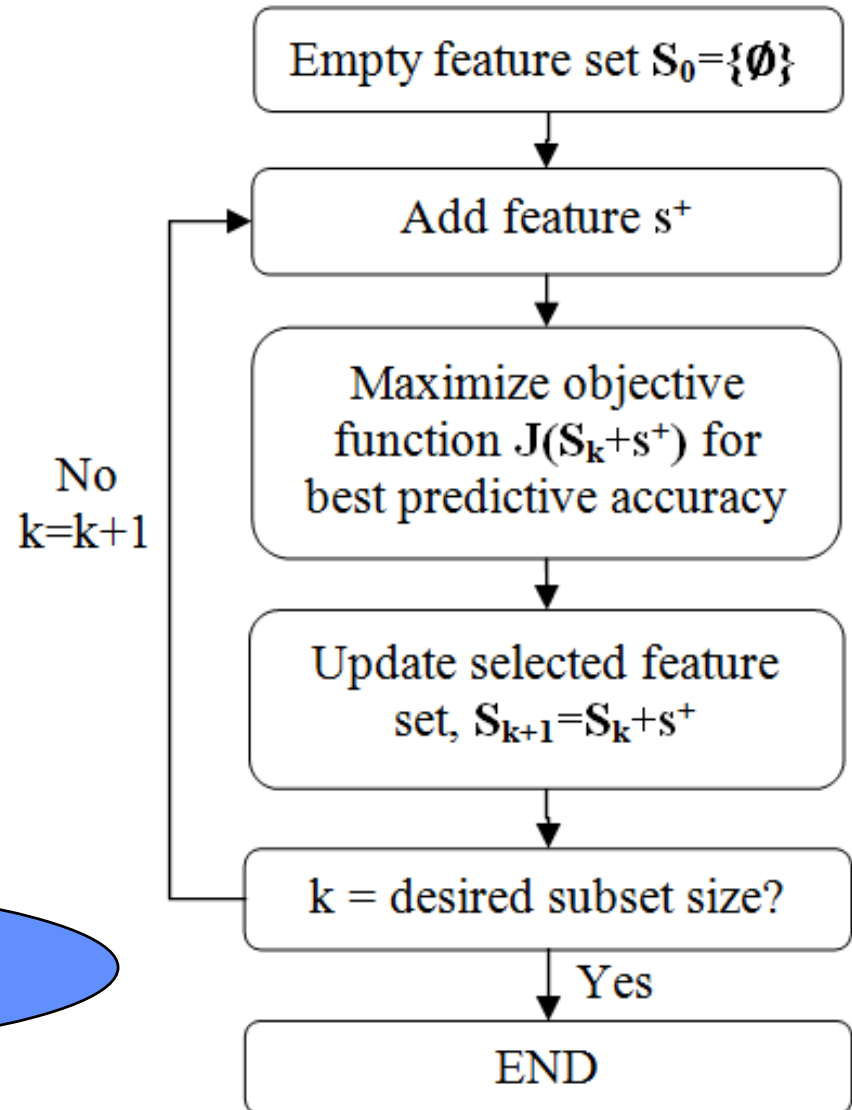
❑ Recurrence features

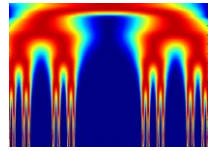
- RR, DET, LMAX, ENT, LAM, TT are extracted for each of the wavelet subseries
- k^{th} level: 2^k number of wavelet subseries
- Selected level: m to n

❑ Total feature size:

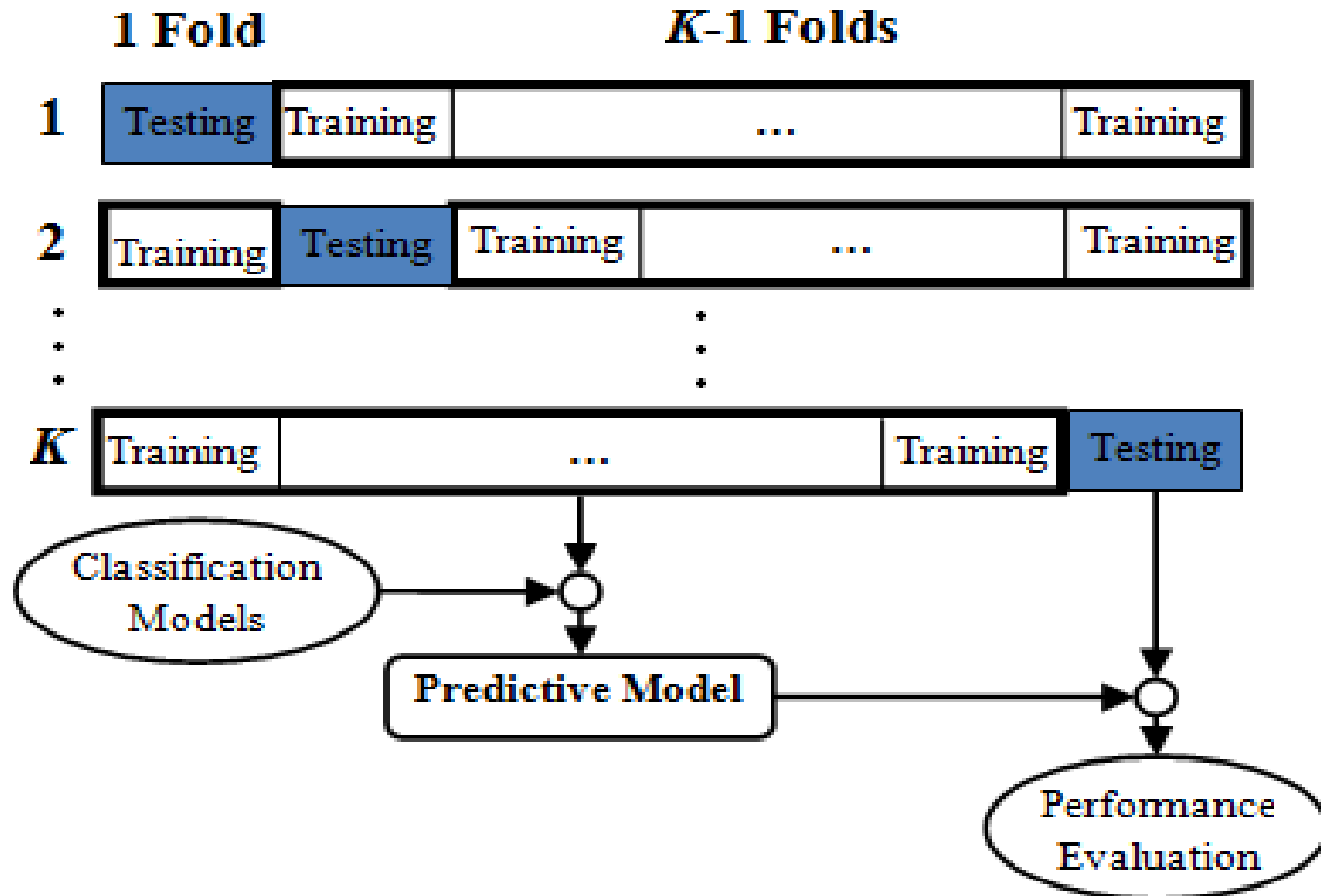
$$\sum_{k=m}^n 6 \times 2^k$$

Curse of dimensionality

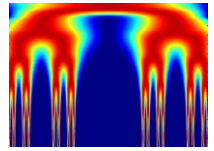




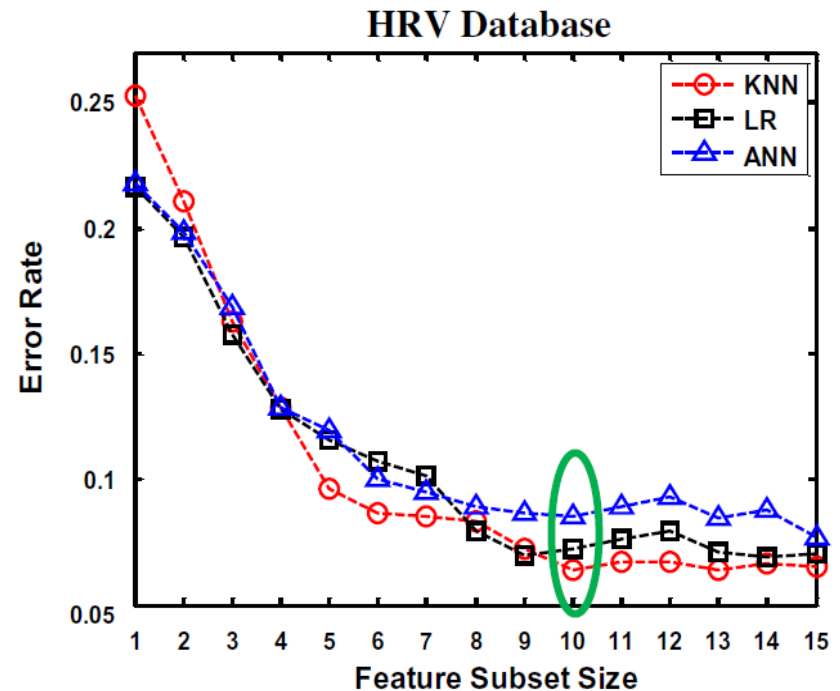
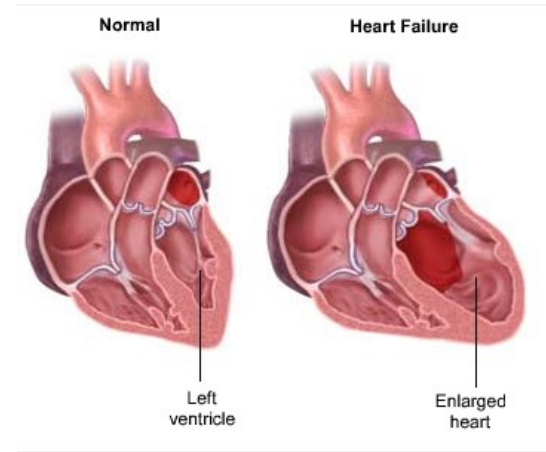
□ K-fold cross-validation & Random sub-sampling



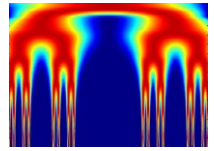
CASE1 – Heart Rate Variability



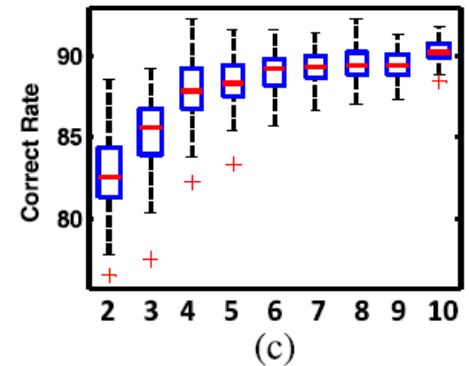
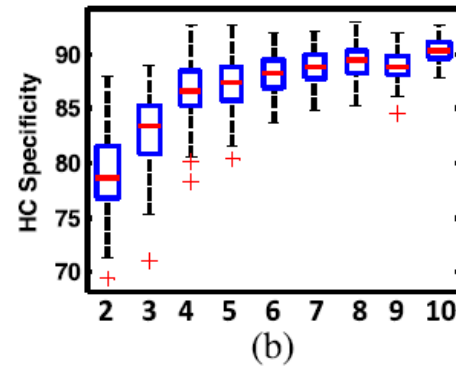
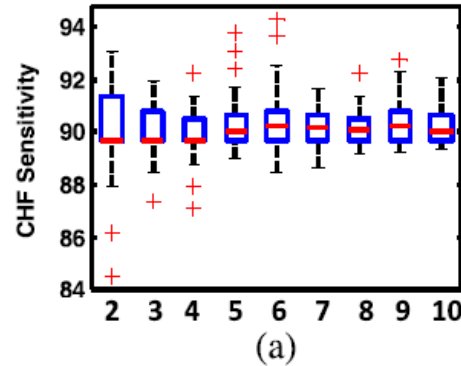
- ❑ Dataset – 24-hour heart rate variability (**HRV**)
 - 54 Health control (**HC**)
 - 29 congestive heart failure (**CHF**)
- ❑ Classification models;
 - K-nearest neighbor (**KNN**)
 - Logistic regression (**LR**)
 - Artificial neural network (**ANN**)
- ❑ Feature selection
 - Selected level: **6** to **9**
 - Total: $\sum_{k=6}^9 6 \times 2^k = 5760$
- ❑ Select **10** features in order to prevent overfitting.



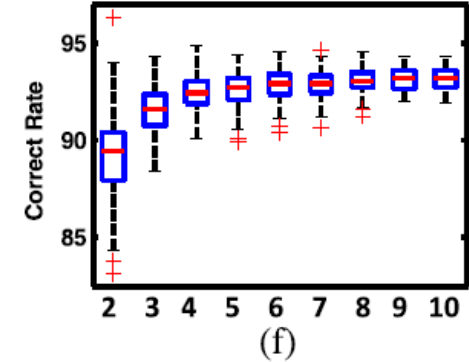
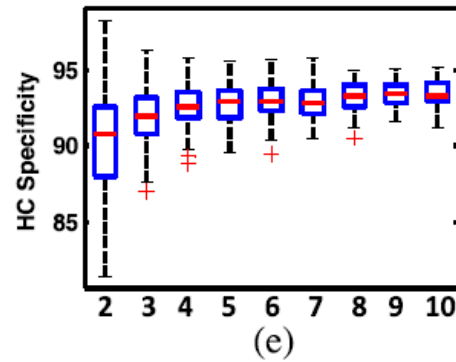
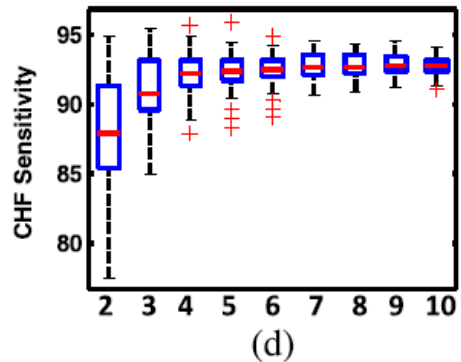
Performance Evaluation (1)



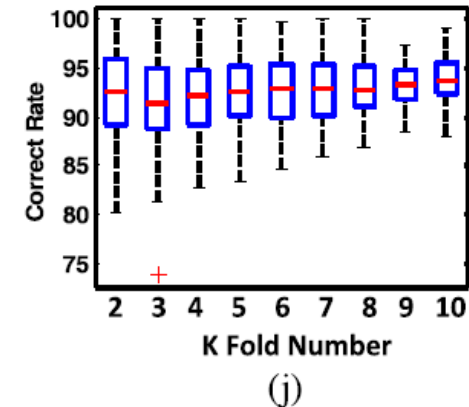
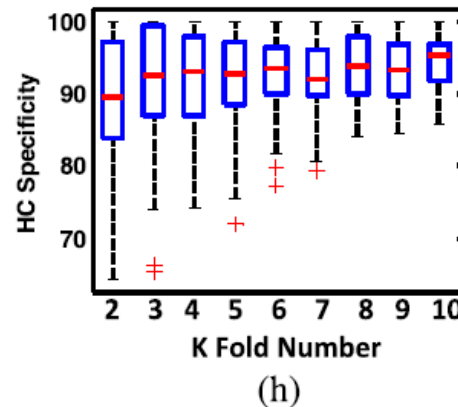
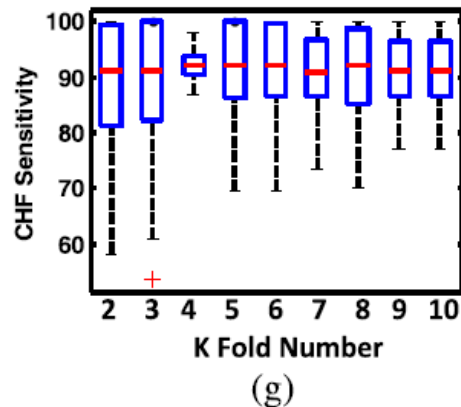
KNN



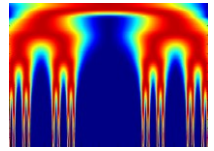
LR



ANN



CASE2 – Vectorcardiogram



❑ Dataset – Vectorcardiogram (VCG)

- 80 Health controls (HC)
- 368 myocardial infarctions (MI)

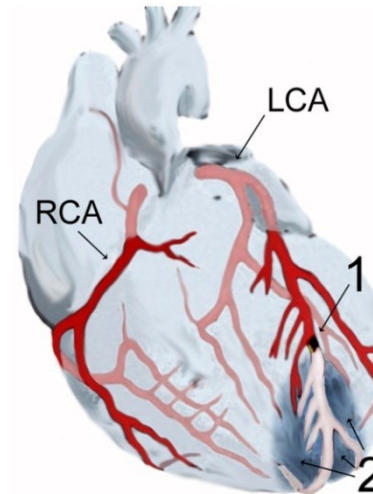
❑ Classification models:

- K-nearest neighbor (KNN)
- Logistic regression (LR)
- Artificial neural network (ANN)

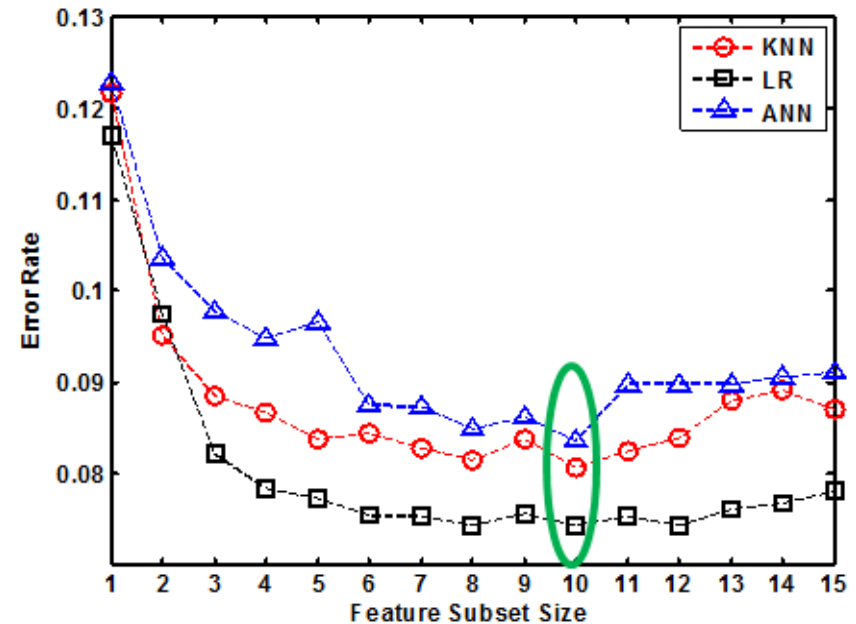
❑ Feature selection

- Selected level: 4 to 5
- Total: $\sum_{k=4}^5 6 \times 2^k = 288$

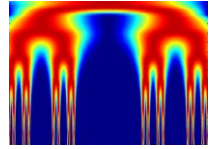
❑ Select 10 features in order to prevent overfitting.



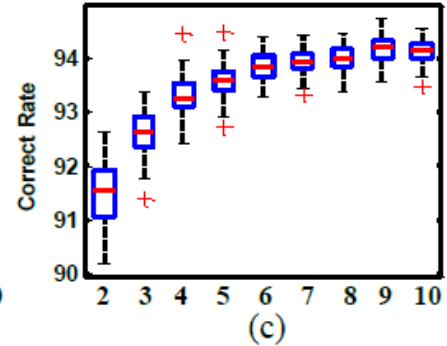
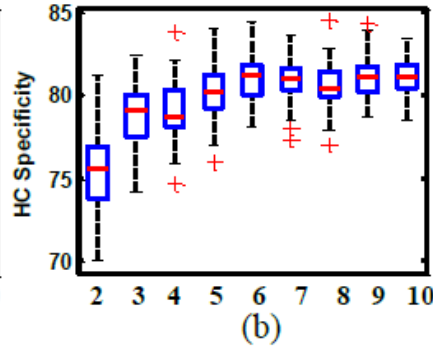
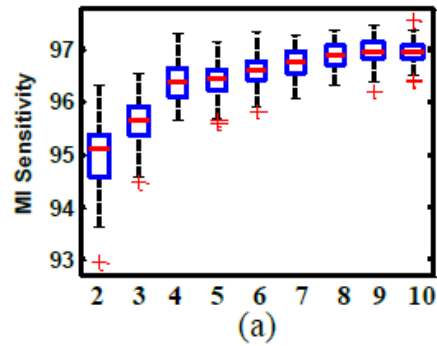
*MI (2), after
occlusion (1)
of a branch of
LCA, RCA*



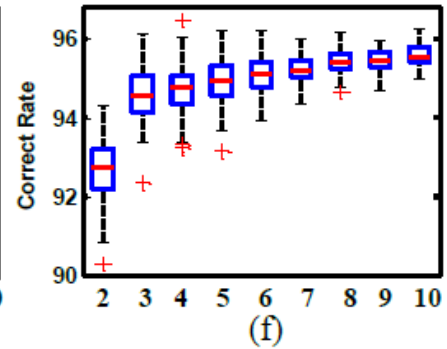
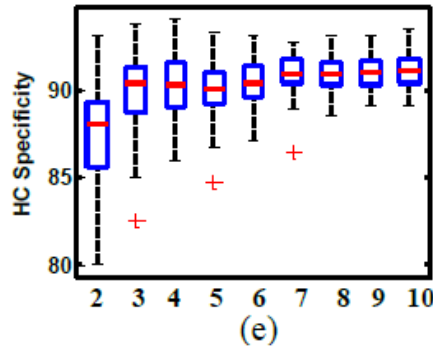
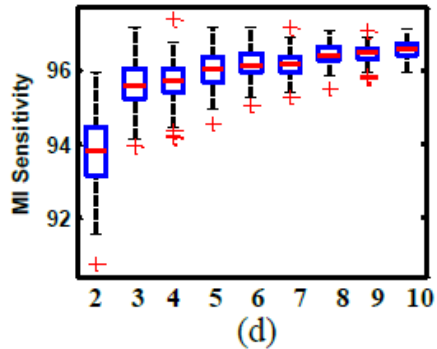
Performance Evaluation (2)



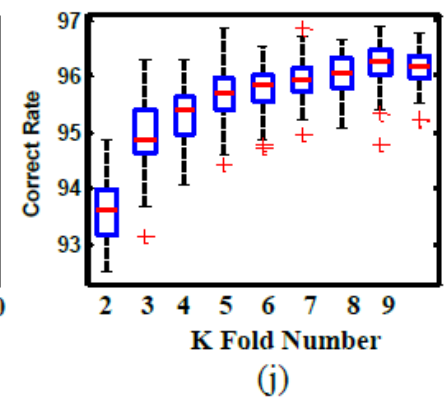
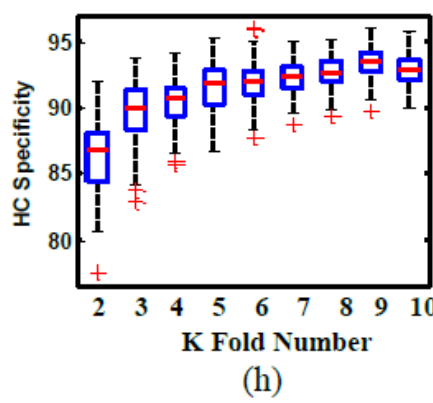
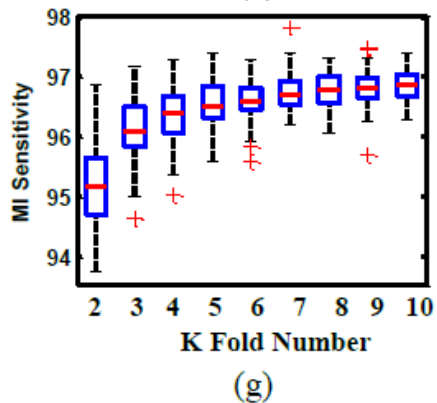
KNN



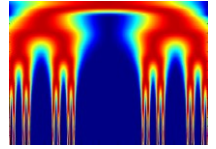
LR



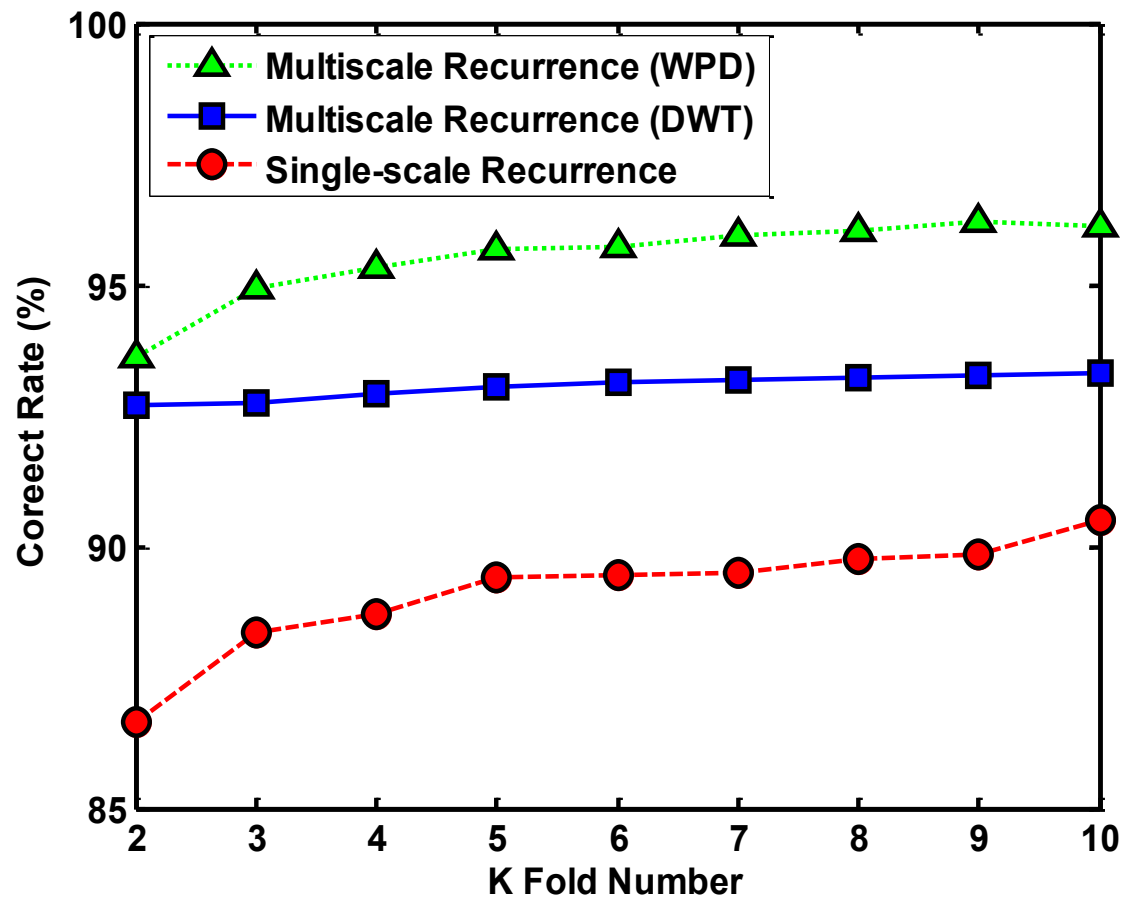
ANN

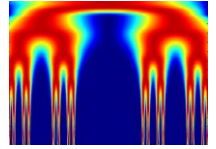


Single-scale VS. Multiscale



- ❑ Comparison of classification correct rates between single-scale and multi-scale (i.e., DWT and WPD) recurrence analysis





❑ Challenges:

- **Big data**
- **Data complexity: nonlinear and nonstationary**

❑ Multiscale recurrence analysis

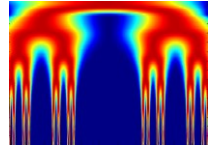
- **Large size of dataset** – dyadic subsampling
- **Nonstationarity** – wavelet decomposition
- **Nonlinearity** – recurrence analysis

❑ Discriminant analysis

- **HRV** database: **92.1%** (sensitivity) and **94.7%** (specificity)
- **VCG** database: **96.8%** (sensitivity) and **92.8%** (specificity)

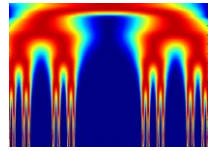
❑ Single-scale vs. multiscale recurrence analysis

Acknowledgements

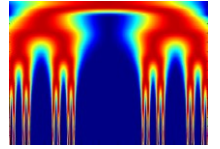


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- ❑ H. Yang, "Multiscale recurrence quantification analysis of spatial cardiac vectorcardiogram (VCG) signals," *IEEE Transactions on Biomedical Engineering*, Vol. 58, No. 2, p339-347, 2011
- ❑ Y. Chen and H. Yang, "Multiscale recurrence analysis of long-term nonlinear and nonstationary time series," *Chaos, Solitons and Fractals*, Vol. 45, No. 7, p978-987, 2012
- ❑ Y. Chen and H. Yang, "Self-organized neural network for the quality control of 12-lead ECG signals," *Physiological Measurement* (**Cover Article**), Vol. 33, No. 9, p1399-1418, 2012
- ❑ H. Yang and G. Liu, "Self-organized topology of recurrence-based complex networks," *Chaos*, under the second revision, Vol. 23, No. 4, 043116, 2013



END
Questions?