Statistics in Volcanology: Uncertainty in Volcanology Data and Models

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Volcanic eruptions...

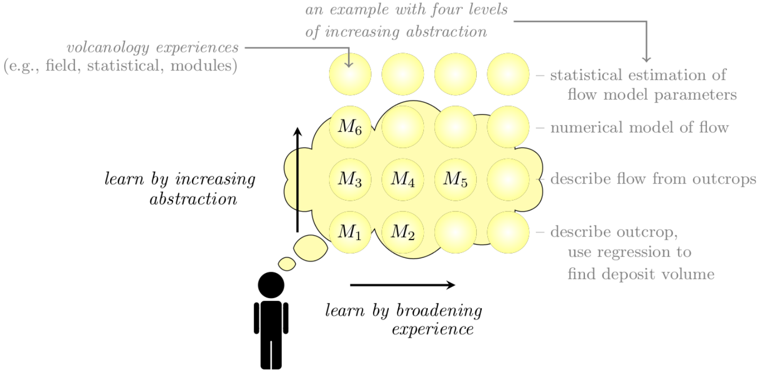
Eruption of Shinmoe-dake volcano, Kirishima volcano complex, Japan.

Major research questions

Tolbachik, Kamchatka, Russia eruption 2013

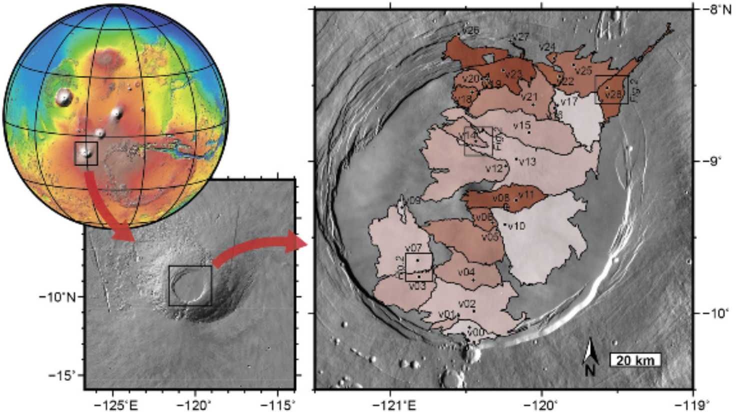
- Forecast the onset, size, duration and hazard of eruptions by integrating observations with quantitative models of magma dynamics.
- Quantify the life cycles of volcanoes globally and overcome our biased understanding.
- Develop a coordinated volcano science community to maximize scientific returns from any volcanic event.

How volcanologists learn about volcanoes...



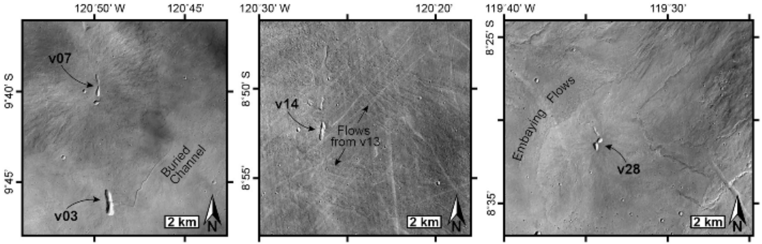
Our schema dictates that we come upon key science questions with a set of prejudgments: an idea of what the problem is, what type of information we are looking for, and what will count as an answer. See Bob Frodeman, 2014 – *Hermeneutics in the field: The philosophy of geology.*

Old volcanism on Mars



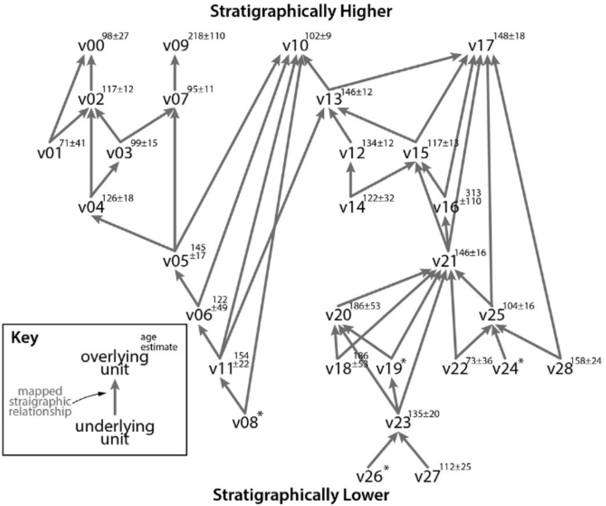
distribution of vents and lava flows in the crater of Arsia Mons

Overlapping lava flows show age relationships



geologists classify images and interpret the relative ages – Steno's Law

A directed graph of age relationships



Ages estimated (with high uncertainty!) from crater density

MC simulation of event rate

Randomly sample ages of all events using directed graph ($M = 10000$ times),
Volcano i of total N formed by event $\hat{e}_i$,
For each set of age estimates, j, for N volcanoes, the cumulative distribution is:

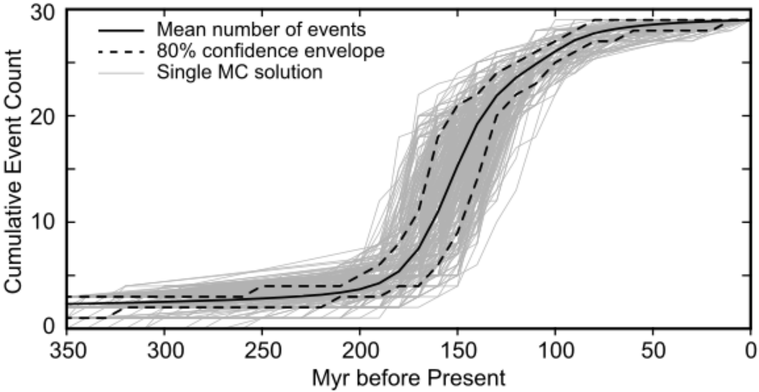
$$X_j(T) = \sum_{i=1}^{N} P\left[\hat{e}_{i,j}, t < T\right]$$

where $P\left[\hat{e}_{i,j}, t < T\right] = 0$ if $T < \hat{e}_{i,j}$ and $P\left[\hat{e}_{i,j}, t < T\right] = 1$ if $T \geq \hat{e}_{i,j}$

$$E(X) = \frac{1}{M} \sum_{j=1}^{M} X_j(T)$$

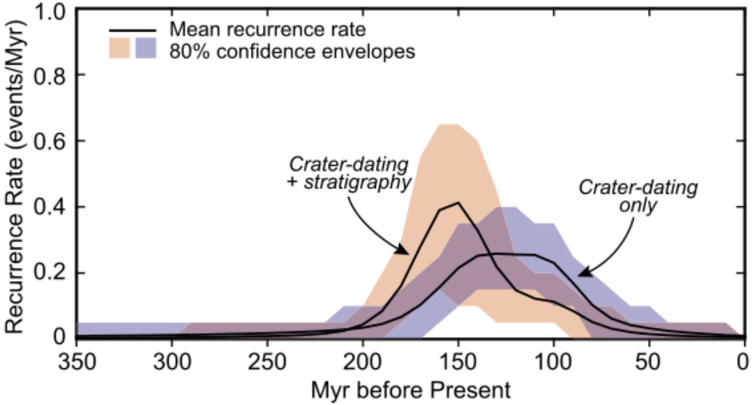
$$R(X) = \frac{\Delta E(X)}{\Delta t}$$

MC simulation of event rate



Based on Monte Carlo simulation using age estimates and stratigraphic information

Estimated distribution of event rate

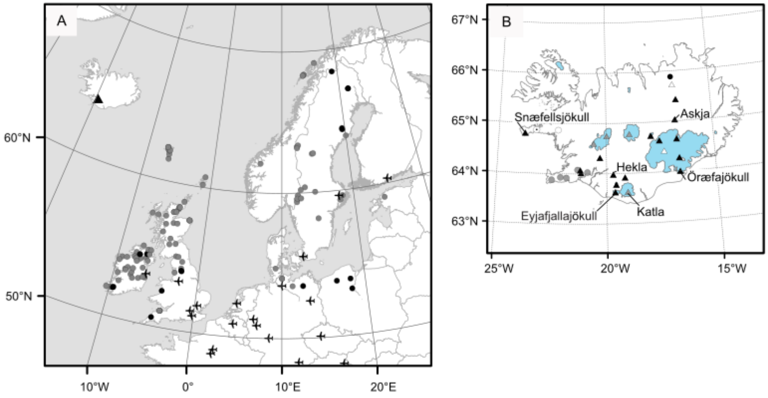


Age distribution of events improved by using directed graph with Monte Carlo simulation

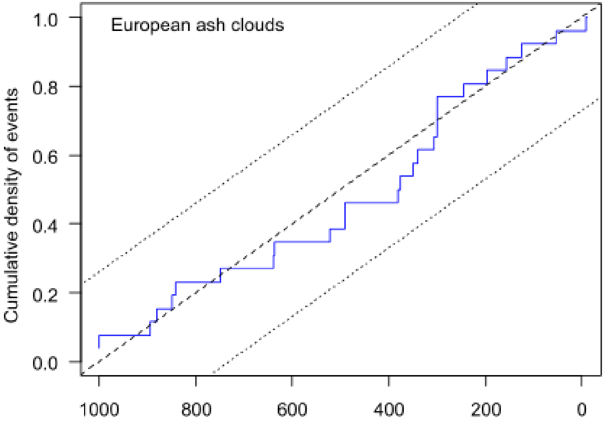
Eyjafjallajökull stops air traffic from North america to Europe

...at a cost of 1 billion euros. How often does this happen?

Volcanic ash preserved in bogs

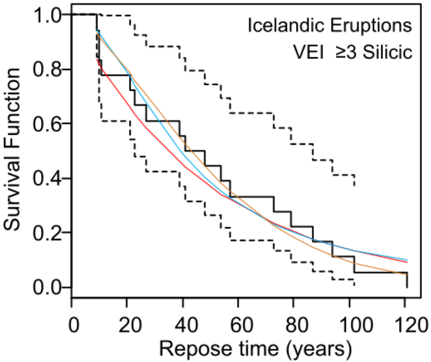


Estimated rate of known events



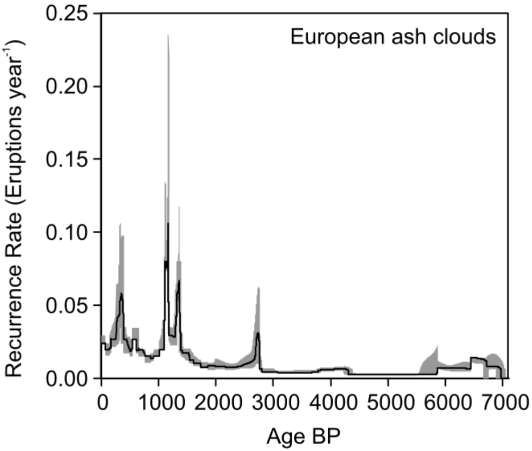
At least the "known" events are stationary in time

Models suggest 44 ± 7 yr



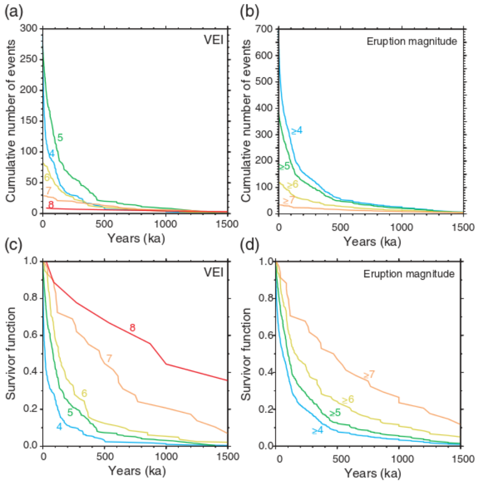
Kaplan–Meier estimate of the survivor function using data from last 1000 yr with fits for various statistical distributions (Weibull, log-logistic, exponential).

Monte Carlo simulation of longer data set



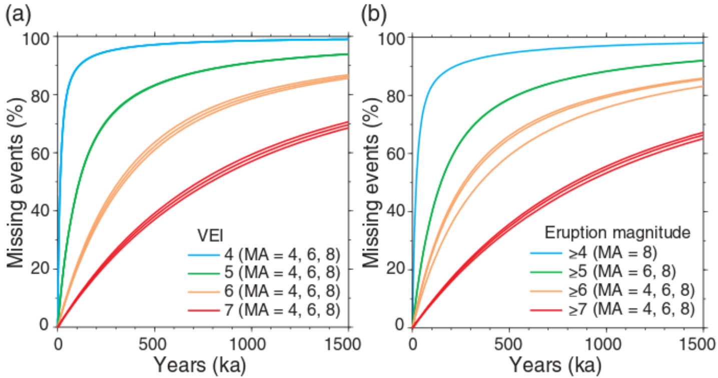
Activity seems to cluster in time over last 7000 yr, average over last 1000 yr may not be representative of true uncertainty.

Missing events in the geologic record



Kiyosugi et al., 2015, How many explosive eruptions are missing from the geologic record? Analysis of the quaternary record of large magnitude explosive eruptions in Japan, Journal of Applied Volcanology

Missing events in the geologic record



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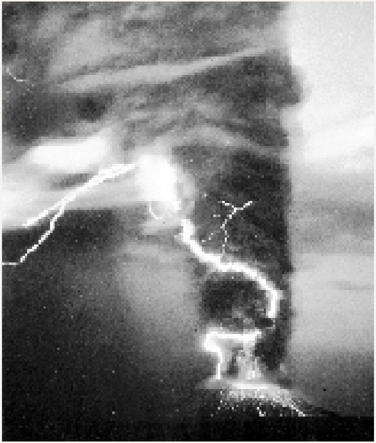
Tephra sedimentation model

Model goal: Estimate the mass erupted

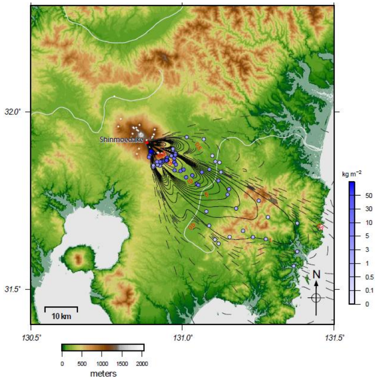


Tolbachik volcano, Russia, 1975

Forward problem: Estimate the accumulation of tephra expected, given volcanic activity.

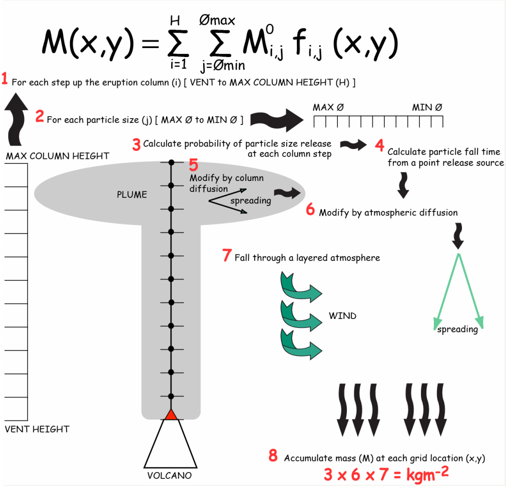
Inverse problem: Given a tephra deposit, what were the eruption parameters that produced this deposit?

Tephra sedimentation model



Tephra sedimentation model

Model algorithm



Tephra2 Model Basics – the implementation

Tephra sedimentation model

Advection – diffusion equation

Single partial differential equation expresses tephra sedimentation

$$\frac{\partial C_j}{\partial t} + w_x \frac{\partial C_j}{\partial x} + w_y \frac{\partial C_j}{\partial y} - v_{l,j} \frac{\partial C_j}{\partial z} = K \frac{\partial^2 C_j}{\partial x^2} + K \frac{\partial^2 C_j}{\partial y^2} + \Phi$$

Expressed dimensionally:

$$\frac{M}{L^3 T} + \frac{L}{T}\frac{M}{L^4} + \frac{L}{T}\frac{M}{L^4} - \frac{L}{T}\frac{M}{L^4} = \frac{L^2}{T}\frac{M}{L^5} + \frac{L^2}{T}\frac{M}{L^5} + \frac{M}{L^3 T}$$

Tephra sedimentation model

ADE

Closed form Eulerian solution to the Advection-Diffusion equation (Suzuki, Macedonio, Lim)

$$f_{i,j}(x,y) = \frac{1}{2\pi\sigma_{i,j}^2} \exp\left[-\frac{(x-\bar{x}_{i,j})^2 + (y-\bar{y}_{i,j})^2}{2\sigma_{i,j}^2}\right]$$

where

$$\bar{x}_{i,j} = x_0 + \sum_{k=0}^{H_i} \frac{w_{x,k} z_k}{v_j, k}$$

and

$$\bar{y}_{i,j} = y_0 + \sum_{k=0}^{H_i} \frac{w_{y,k} z_k}{v_j, k}$$

Tephra sedimentation model

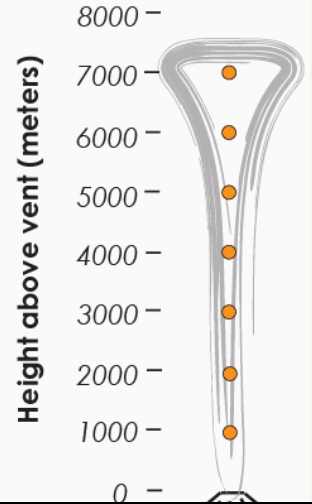
ADE

Variable Reynold's number for particle settling (Bonadonna et al., 1998)

$$v_j = \begin{cases} \dfrac{\rho_j g d_j^2}{18\mu} & \text{if laminar, } Re < 6, \\ d_j \left[\dfrac{4g^2\rho_j^2}{225\mu\rho_a}\right]^{1/3} & \text{if intermediate, } 6 \leq Re < 500, \\ \left[\dfrac{3.1\rho_j g d_j}{\rho_a}\right]^{1/2} & \text{if turbulent, } Re \geq 500, \end{cases}$$

Tephra sedimentation model

Plume Geometry



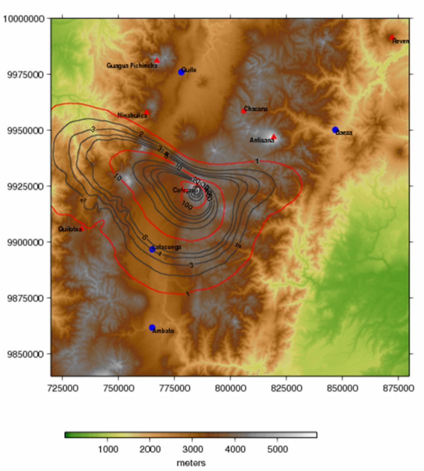
$$t_{i,j} = \sum_{k=0}^{H_i} \frac{z_k}{v_j}$$

$$t_i^{'} = \left[0.2 h_i^2\right]^{2/5}$$

$$\sigma_{i,j}^2 = \begin{cases} 4K(t_{i,j} + t_i^{'}) & \text{if } t_{i,j} < \tau, \\ \frac{8C}{5}(t_{i,j} + t_i^{'})^{5/2} & \text{if } t_{i,j} \geq \tau, \end{cases}$$

Tephra sedimentation model

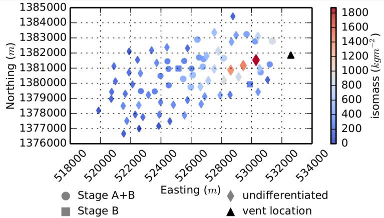
Example forward model results



- Maximum Column Height: 14000 m
- Total Mass: 1×10^{11} kg
- Median Grain Size: $0\,\phi$
- STD Grain Size: $1\,\phi$
- Wind from NOAA reanalysis

Kirishima 2011

PEST Model Inversion

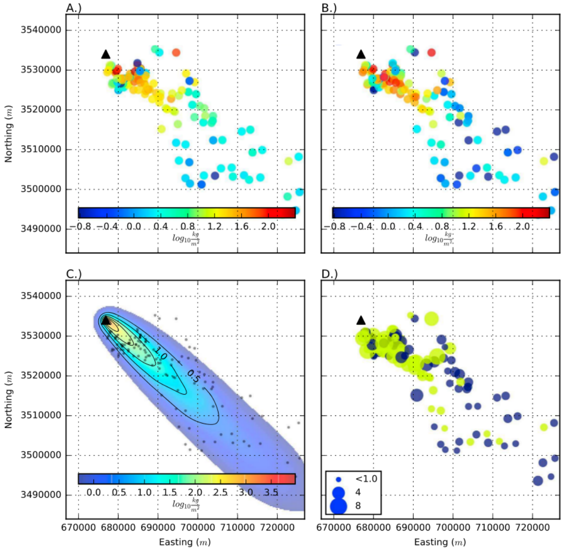


try modeling 1992 eruption stages (A and B), and using grain size data collected from each sample pit

Use the PEST inversion method to interpret the 2011 Kirishima eruption source parameters:

- Singular value decomposition with Tikhonov regularization
- Bayesian procedure - specify prior information and output pdf of parameter model
- Using the open source, open access PEST code (Parameter Estimation, SVD, and Tikhonov)

PEST Model Inversion



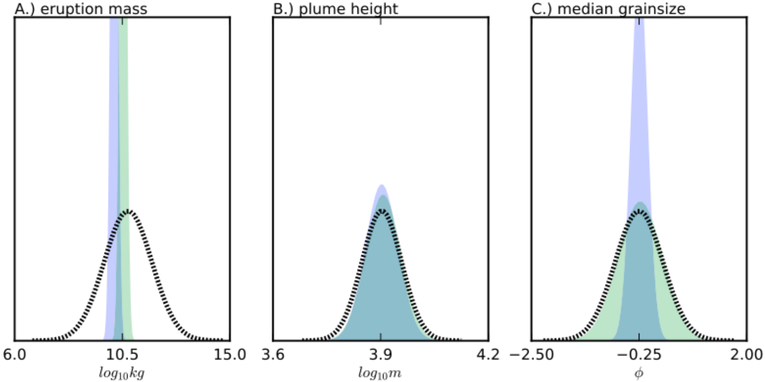
PEST Model Inversion

Table 1. Prior and Posterior Eruption Parameter Summary for the 2011 Kirishima-Shinmoedake Eruption for Inversion Using Only Tephra Isomass Data and Inversion Using Tephra Isomass and Grain Size Data Together[a]

		Prior		Isomass Only Posterior		Isomass and Grain Size Posterior	
Parameter	Transform	Mean	σ	Mean	σ	Mean	σ
Eruption mass (kg)	$\log_{10}$	10.7404	1.0000	10.5337	0.1238	10.1650	0.1265
Plume height (m)	$\log_{10}$	3.9031	0.0555	3.9069	0.0491	3.9029	0.0459
Median grain size (ϕ)	none	−0.2500	0.5000	−0.2255	0.4645	−0.2450	0.1729
Grain size σ (ϕ)	$\log_{10}$	0.3010	0.1193	0.3245	0.1107	0.3044	0.0350
Coarse grain density (kg m^{-3})	$\log_{10}$	3.0000	0.0440	2.9422	0.0438	3.0000	0.0436
Fine grain density (kg m^{-3})	$\log_{10}$	3.4150	0.0167	3.3802	0.0167	3.4150	0.0167
Fall time threshold (s)	$\log_{10}$	2.0000	1.0000	2.0097	1.0000	2.0000	1.0000
Diffusion (m^2 s^{-1})	$\log_{10}$	3.0000	0.8693	2.8069	0.8693	3.0000	0.8693
α-β ratio (dimensionless)	$\log_{10}$	0.0000	0.7500	−1.7824	0.7493	−0.1880	0.2469
Eddy constant (m^2 s^{-1})	$\log_{10}$	−1.3979	0.4886	−1.3950	0.1867	−1.3720	0.1286

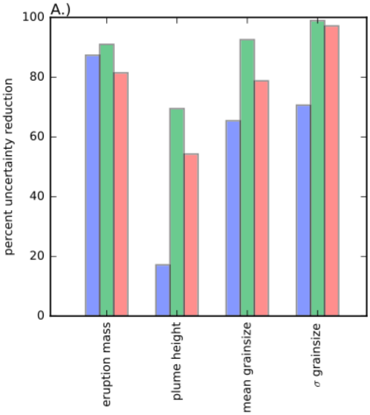
[a]Mean and uncertainty (1σ) are shown. To find range, add and subtract 1σ or 2σ (for 95% confidence interval) from the mean and apply the transform.

PEST Model Inversion



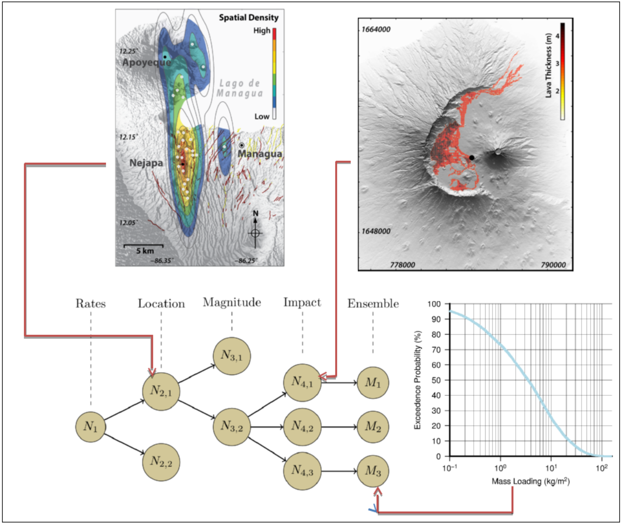
Prior (dashed) versus posterior (shaded) parameter estimates

PEST Model Inversion



Uncertainty in eruption source term parameters seems to be reduced using the PEST inversion and Tephra2 forward model.

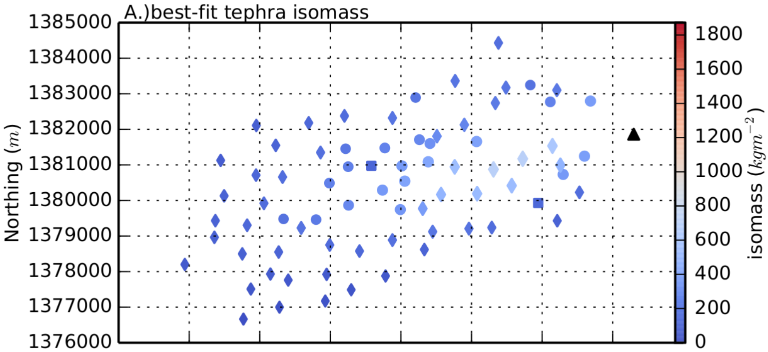
A logic tree for forecasts



Conclusions

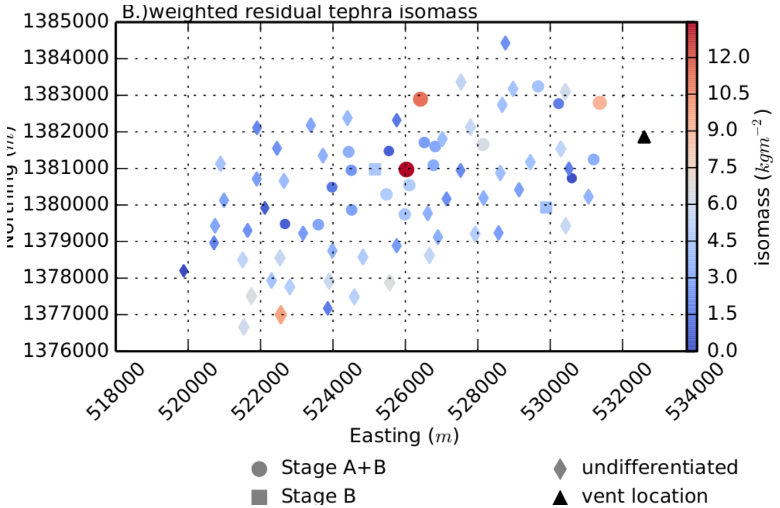
- Volcano science is about using observations (data) to improve our models of the timing of volcanic eruptions, their magnitudes, and potential impacts.
- High uncertainty because volcanoes are difficult to observe, erupt infrequently and exhibit a huge range of behaviors.
- Great opportunity for the application of statistical methods in interpretation of past events and forecasting future events.

PEST Model Inversion



estimated model

PEST Model Inversion



model residuals

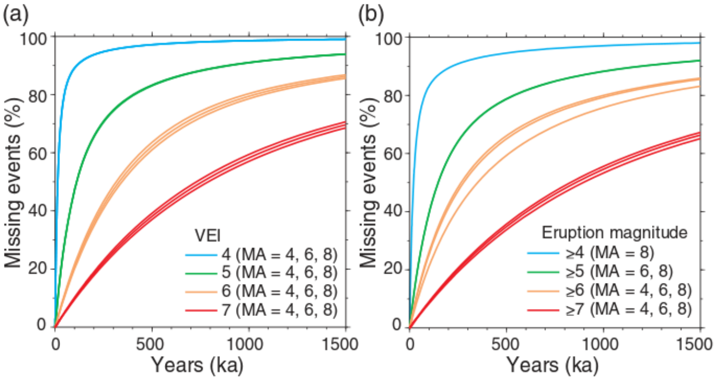
PEST Model Inversion

	Parameter	Value		Prior 95% bounds		posterior 95% bounds	
stage	(units)	initial	best-fit	lower	upper	lower	upper
A	Fickian diff. (m^2s^{-1})	1000.000	335.977	1.414	7.071E+05	250.839	450.013
A	alpha-beta ratio ()	1.000	1.213	0.0316	31.623	0.755	1.949
A	eddy diff. (m^2s^{-1})	0.0050	0.0050	0.0017	0.0150	0.0017	0.0150
A	eruption mass (kg)	5.500E+10	9.681E+09	1.230E+10	2.460E+11	8.088E+09	1.159E+10
A	fall time threshold (s)	3.000E+03	3.000E+03	1133.893	7.937E+03	1133.893	7.937E+03
A	grainsize std. dev. (ϕ)	0.500	1.393	0.0177	14.142	1.218	1.594
A	lithic density (kgm^{-3})	2.600E+03	3.000E+03	2.350E+03	2.877E+03	2.716E+03	3.313E+03
A	median grainsize (ϕ)	-0.500	-0.757	-2.500	1.500	-0.969	-0.545
A	plume height (km)	7.000E+03	6.922E+03	3.834E+03	1.278E+04	6.070E+03	7.894E+03
A	pumice density (kgm^{-3})	1000.000	757.954	780.189	1281.740	595.588	964.584
B	Fickian diff. (m^2s^{-1})	1000.000	1.536E+04	1.414	7.071E+05	8.105E+03	2.910E+04
B	alpha-beta ratio ()	1.000	1.767	0.0316	31.623	1.279	2.440
B	eddy diff. (m^2s^{-1})	0.0050	0.0050	0.0017	0.0150	0.0017	0.0150
B	eruption mass (kg)	1.000E+10	3.312E+10	1.155E+09	8.660E+10	1.664E+10	6.593E+10
B	fall time threshold (s)	3.000E+03	3.000E+03	1133.893	7.937E+03	1133.893	7.937E+03
B	grainsize std. dev. (ϕ)	0.500	1.206	0.0177	14.142	0.988	1.472
B	lithic density (kgm^{-3})	2.600E+03	2.788E+03	2.350E+03	2.877E+03	2.520E+03	3.085E+03
B	median grainsize (ϕ)	-0.500	0.363	-2.500	1.500	-0.147	0.873
B	plume height (km)	3.000E+03	3.431E+03	1224.745	7.348E+03	1716.833	6.857E+03
B	pumice density (kgm^{-3})	1000.000	899.676	780.189	1281.740	703.482	1150.587

estimated parameters

Missing events in the geologic record

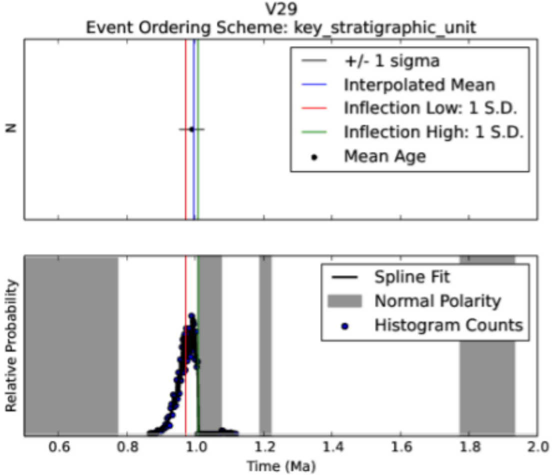
Statistics in Volcanology
Chuck Connor
Arsia Mons
Eyjafjallajökull
Uncertainty in Recurrence Rate
Estimating eruption magntiude
Forecasts
Conclusions



Kiyosugi et al., 2015, How many explosive eruptions are missing from the geologic record? Analysis of the quaternary record of large magnitude explosive eruptions in Japan, Journal of Applied Volcanology

A Volcanic Event Age Model (VEAM)

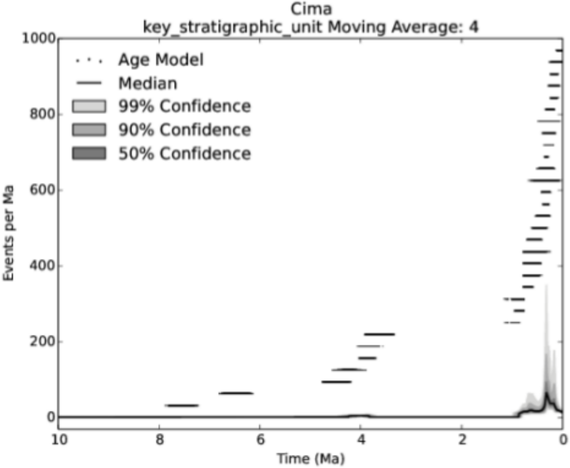
Age estimate of one lava flow in the Cima Volcanic field



Wilson, Richardson and others

A Volcanic Event Recurrence Rate Model (VERRM)

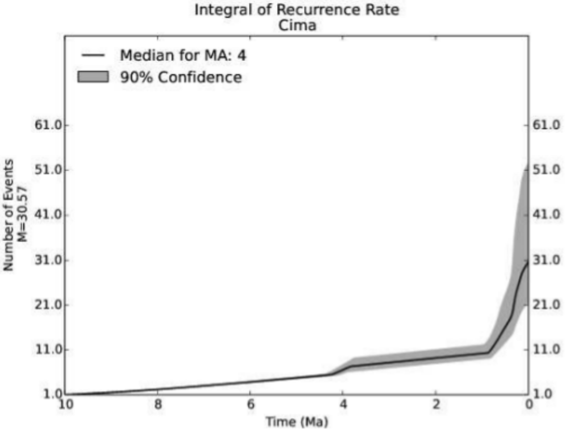
An age model for volcanic events in the Cima Volcanic Field



Wilson, Richardson and others

A Volcanic Event Recurrence Rate Model (VERRM)

A reccurrence rate model for the Cima volcanic field, emphasizing uncertainty in current event rates



Wilson, Richardson and others